

KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN  
TE AMSTERDAM

PROCEEDINGS

VOLUME XXX

Nº. 2

President: Prof. F. A. F. C. WENT

Secretary: Prof. L. BOLK

(Translated from: "Verslag van de gewone vergaderingen der Afdeling  
Natuurkunde", Vols. XXXV and XXXVI)

CONTENTS

- M. PINKHOF: "Contribution to the Explanation of Complex Halos". (Communicated by Prof. E. VAN EVERDINGEN), p. 172.
- H. ZWAARDEMAKER: "Radiation-substances and Cardiac Hormones", p. 184.
- J. BOESEKEN and S. L. LANGEDIJK: "The Light Oxydation of Alcohols as Contribution to the Knowledge of the Photo-Chemical Phenomena", p. 189.
- IDA LUYTEN: "On the favorable effect of 35° C. on the cell-formation in foliage-leaves of *Hyacinthus orientalis*". (Communicated by Prof. A. H. BLAAUW), p. 197.
- A. H. BLAAUW: "On the atmospheric humidity during the flower-formation of the *Hyacinth*", p. 206.
- E. BORTOLOTTI: "On metric connexions with absolute parallelism". (Communicated by Prof. JAN DE VRIES), p. 216.
- R. WEITZENBÖCK: "Ueber Syzygien bei sechs binären Linearformen", p. 219.
- S. DE BOER: "The action of veratrin on skeletal muscles. At the same time an application of our knowledge of the segmentation of the frog's *M. gastrocnemius* on the problem of the refractory stage of skeletal muscles after veratrin poisoning. (Communicated by Prof. R. MAGNUS), p. 235.
- J. J. VAN LAAR: "On the Course of the Melting-Point Line of Helium at Very Low Temperatures". (Communicated by Prof. H. A. LORENTZ), p. 244.
- F. M. JAEGER, H. G. K. WESTENBRINK and F. A. VAN MELLE: "Roentgenspectrographic Investigations on the Structure of the Artificial Ultramarines and the Problem concerning their Relations to the Minerals Hauyne, Nosean, Sodalite, Lazurite and Nephelite", p. 249.
- JAN DE VRIES: "A complex of Conics", p. 268.
- H. A. BROUWER: "Fractures and faults near the surface of moving geanticlines. IV. The elastic rebound of the earth crust in Central-Honshu", p. 272.
- J. A. SCHOUTEN: "The invariants of linear connexions with different transformations". (Communicated by Prof. HENDRIK DE VRIES), p. 276.
- F. A. H. SCHREINEMAKERS: "Equilibria in systems in which phases, separated by a semi-permeable membrane" XIX, p. 282.
- H. BOSCHMA: "On the Larval Forms of *Rhizocephala*". (Communicated by Prof. C. PH. SLUITER), p. 293.
- J. MÜLLER: "On the occurrence of Vascularized Enamel-organs. (Communicated by Prof. L. BOLK), p. 298.
- H. R. HOOGENRAAD: "On the structure, life-history and development of *Hedrocystis pellucida* HERTW. & LESS. (Communicated by Prof. J. F. VAN BEMMELEN), p. 308.
- L. BOLK: "On the Origin of human Races", p. 320.
- MA (WEN CHAO): "A comparison of the form of the callosum and septum in Chinese. Philippino- and Dutch brains". (Communicated by C. U. ARIENS KAPPERS), p. 329.

12

**Meteorology.** — *Contribution to the Explanation of Complex Halos.*  
By M. PINKHOF. (Communicated by Prof. E. VAN EVERDINGEN)

(Communicated at the meeting of June 26, 1926)

1. *Introduction.*

Though at present most of the separate halo phenomena have been explained in a satisfactory way, i. e. have been brought in connection with ice crystals of acceptable shape and orientation, it must be admitted that as yet comparatively little attention has been given to the theory of complex halos. The second edition of PERNTER's *Meteorologische Optik* revised by EXNER, does not yet deal with it. Nevertheless it is necessary also for the theory of the simple halo phenomena, to study the complex halos more closely. By the simultaneous appearance of different phenomena an explanation of one of them, based on the occurrence of very specially formed crystals, can be rendered less probable. It is beyond doubt that, in the case of complicated halos, there is very often a greater number of the component parts mutually related than could be contributed to by one and the same crystal at a fixed moment. In the latter case WEGENER (20) — who was the first to occupy himself with the complex halos — speaks of "verschwisterte Halos". All the other phenomena that make their appearance simultaneously, he calls "vergesellschaftet". I myself have pointed out afterwards (17, p. 66) that WEGENER's "vergesellschaftete Halos" should be divided in their turn into two groups:

1. phenomena arising in the same cloud;
2. phenomena that find their origin in different Ci or Ci-St clouds present at the same time.

As a classification into two, I should therefore prefer a group A of phenomena in one cloud ("verschwistert" in a wider sense) and a group B of phenomena in different clouds ("vergesellschaftet" in a narrower sense). In what follows here, an attempt is made to account for the way in which the phenomena of group A can be connected together.

On an earlier occasion I made an attempt to find a solution of this problem of the complex halos (16, p. 72; 17, p. 65). I started from the supposition that in one ice-cloud, arising as it does under very definite circumstances, a wide diversity is not possible either in the shape of the crystals, which is dependent on the temperature, or in their size, which is related with the vapour tension. From this there ensues amongst others that the simultaneous presence of the two fundamental shapes of ice crystals — the *plates* falling with vertical principal axes, and the



rods falling with horizontal principal axes — is not probable. Basing myself on DOBROWOLSKI's experiences (6), I drew, however, attention to the fact that, in consequence of the difference in size of their central cavity, some of the rods of the most prevalent form, i.e. the hemimorphous form, will fall with vertical, others with horizontal principal axes, (Fig. 1).

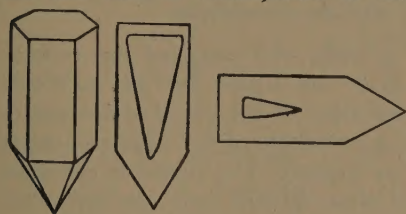


Fig. 1. Hemimorphous ice prisms.

Accordingly, if a cloud consists merely of hemimorphous prisms, a pretty complex halo can already appear in this way:

1. Ordinary ring caused by the practically never lacking not specially directed crystals;
2. A. Parhelia } caused by crystals with vertical principal axes.  
B. Circumzenithal arc }
3. Upper and lower tangent arc, due to crystals with horizontal principal axes.

Theoretically the following phenomena might be added to this:

- large ring ("verschwistert" with 1);
- arcs of Lowitz ("verschwistert" with 2);
- parhelic ring ("verschwistert" with 2 or 3);
- light column ("verschwistert" with 2 or 3);
- lateral tangent arcs to the large ring ("verschwistert" with 3).

From the fact, however, that these phenomena are much rarer than the first mentioned, there follows that they evidently require still other circumstances than those under which the general form of a complex halo so often occurs.

In my earlier publications (16, 17) I have not occupied myself with the very rare halo forms; HASTINGS, however, has tried to draw up a theory (11), in which also the possibility of their presence was taken into consideration. I do not, however, consider his attempt as successful:

1. because he uses for his explanation at the same time the two fundamental forms: plates and rods;

2. because for some of the phenomena he does not only require an orientation in a definite plane of the crystallographic principal axis, but also a fixed position for definite lateral faces.

However this may be, it remains necessary, in an explanation of complex halos, to take the less usual phenomena into account as well. The question then arises: under what circumstances can hemimorphous iceprisms cause, besides the more common phenomena, the rarer ones also? To these belong not only the above-mentioned phenomena, but also the *anthelion* and the *paranthesis*, of which it is not even certain with what other halo forms they are "verschwistert". Before proceeding

to the discussion of this question, it is desirable to give a summary of the explanations of the anthelion c. a. published so far.

## 2. *The Anthelion and the Paranthelia.*

BRAVAIS (5, p. 189, cf. also 14, p. 424), who imagined the crystals, when falling, to orientate themselves in such a way that they meet with the least resistance, explained the anthelion by the aid of double interior total reflection against planes making an angle of  $90^\circ$  with each other (Fig. 2) and which are in a vertical position. This theory is untenable since it has become known that ice plates fall with their bases *horizontal*, and ice rods — supposing they do not rotate about their principal axes in falling — will not have an edge downward, but a side face, in consequence of which the side face corresponding with A of Fig. 2 will not be vertical.

Of the explanations of the paranthelia before BESSON's, that of SORET (cf. 14, p. 428) was considered as the least artificial (Fig. 3). Several

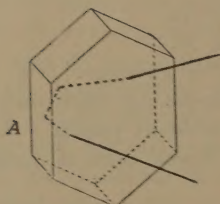


Fig. 2. BRAVAIS' explanation of the anthelion. (According to BESSON).

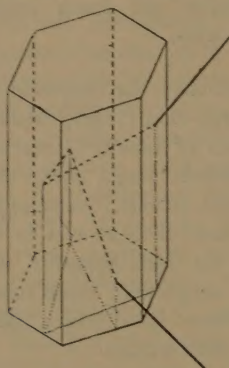


Fig. 3. SORET's explanation of the paranthelia. (According to BESSON).

objections, however, were advanced against it by BESSON (2, p. 77), and EXNER (14, p. 428).

As a substitute for the earlier theories BESSON (1; 2 p. 80) has given an explanation, according to which the anthelion and the paranthelia are due to aggregates of crystals. To the figures 4 and 5 nothing need be added to make it understood that the crystal faces which act as "double mirrors", <sup>1)</sup> at an angle of  $90^\circ$  always change the direction of the horizontal projection of the rays  $180^\circ$ , and at an angle of  $120^\circ$  give it a deviation of  $120^\circ$ .

By the aid of the aggregates of figures 4 and 5 the phenomena in question are therefore, "easily" explained.

<sup>1)</sup> Mirrors including an angle  $\alpha$ , reflect the light in a direction deviating  $360^\circ - 2\alpha$  from the original, independent from the angle of incidence. German: Winkelspiegel.

Now the question is: 1. do the postulated aggregations occur in reality, and even in large quantities? 2. do they bear their reflecting side faces really vertically?

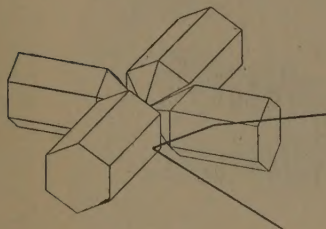


Fig. 4. BESSON's explanation of the anthelion. (According to BESSON).

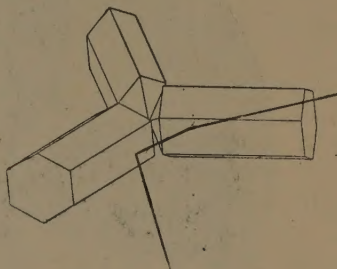


Fig. 5. BESSON's explanation of the paranthelia. (According to BESSON).

In answer to the first question it may be said that the very basis on which BESSON founds his theory is DOBROWOLSKI's experience (6) that the hemimorphous ice rods very often form radiary aggregates by uniting their tips. In 1903 already, DOBROWOLSKI gave a number of drawings of such aggregates in his extensive publication, both of completed ones and of those in statu nascendi (6, p. 32—37).

Not only these aggregates of 3 and 4 arise by union of hemimorphous ice rods. DOBROWOLSKI showed that also the so-called holohedric prisms have, after all, been formed by two joined hemimorphous ones. In 1916 DOBROWOLSKI (7) was able to publish micro-photographs of all these forms, made by F. HALLBERG. Four of these photos are reproduced in the figures 6—9<sup>1)</sup>.

The answer to the second question cannot be given as yet with certainty. As may be seen from the photo, it is not to be avoided that the edges of the crystals soon melt off during the observation, so that the exact position of the faces cannot be ascertained. BESSON (4, p. 379), however, points out that from the fact that in the combinations of 2 the faces always lie exactly in each other's prolongation (twisted specimens have never been observed), it may be concluded that also with combinations of 3 or 4, the mutual position of the component crystals shows great regularity. In this case they would both turn either a face or an edge towards each other. If it is a face, they act as "double mirrors" with vertical planes.

In his treatise of 1923 BESSON complains that his theory of the anthelion c.a., published as early as 1907, has not been favourably received. "It has been thought improbable that shapes so complicated

<sup>1)</sup> I am greatly indebted to the Société Astronomique de France for the loan of the clichés and I gladly express my thanks to Dr. BESSON for his kind assistance.



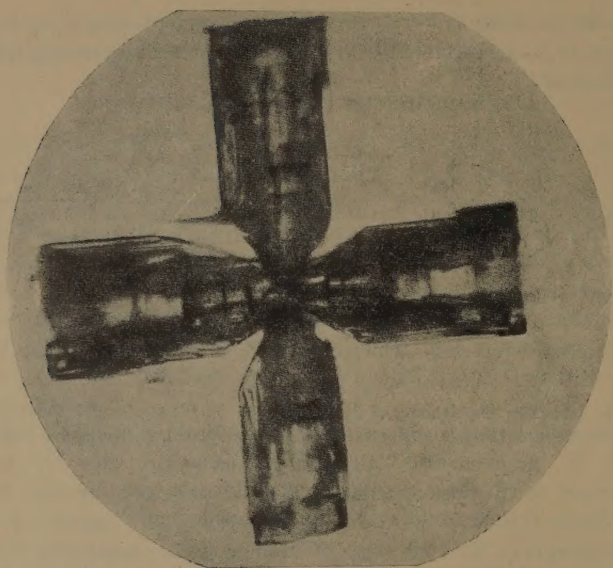


Fig. 6. Aggregate of 4 hemimorphous ice prisms (Photo F. HALLBERG).  
(Cliché of the Société Astronomique de France).

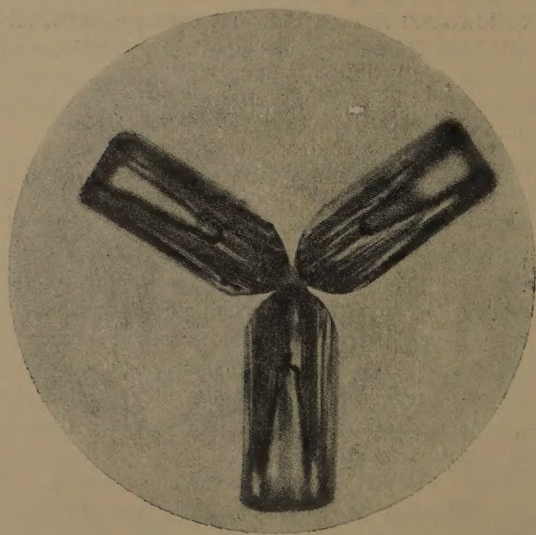


Fig. 7. Aggregate of 3 hemimorphous ice prisms (Photo F. HALLBERG).  
(Cliché of the Société Astronomique de France).

could exist in the atmosphere in quantities large enough to give rise to a luminous phenomenon". The microphotos of HALLBERG reproduced

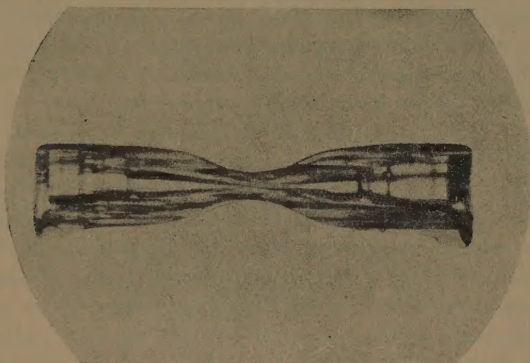


Fig. 8. Aggregate of 2 hemimorphous ice prisms in statu nascendi (Photo F. HALLBERG)  
(Cliché of the Société Astronomique de France).

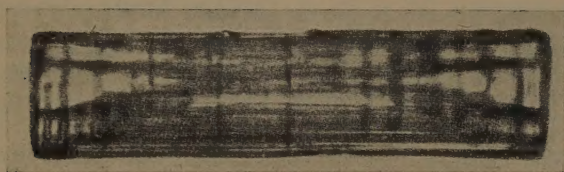


Fig. 9. Aggregate of 2 hemimorphous ice prisms (holohedric prism). (Photo F. HALLBERG).  
(Cliché of the Société Astronomique de France).

here show irrefutably that radiary aggregates of ice crystals exist and are well defined crystalline forms.

It seems to me that the complex halo, observed by me at Amsterdam on December 23<sup>rd</sup> 1925, and described in the March number 1926 of "Hemel en Dampkring" (19), supplies an important support to BESSON's theory. At the same time the explanation of complex halos can get nearer to a satisfactory solution.

### 3. The Halo of Dec. 23, 1925.

A. The following phenomena were observed:

- |                                 |                                      |
|---------------------------------|--------------------------------------|
| 1. ordinary ring                | faint                                |
| 2. upper tangent arc to same    | exceedingly bright and extensive     |
| 3. parhelia with arcs of LOWITZ | bright                               |
| 4. large ring                   | relatively very bright and extensive |
| 5. upper tangent arc to same    | relatively faint                     |
| 6. anthelion                    | faint uncoloured little column.      |

N<sup>o</sup>. 1—5 are represented diagrammatically in Fig. 10, which has been borrowed from the above-mentioned article in "Hemel en Dampkring", to which we refer for the description of the course of the phenomenon.<sup>1)</sup>

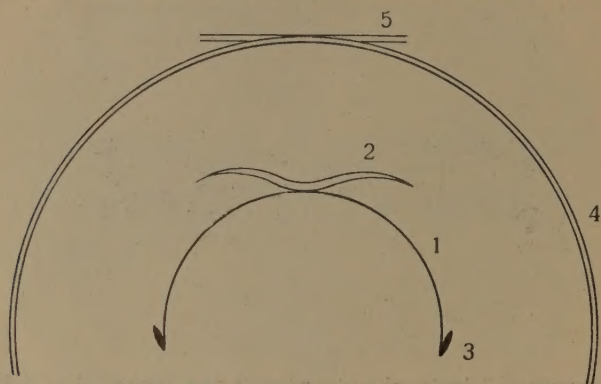


Fig. 10. Sketch of the halo at Amsterdam on Dec. 23, 1925.

1 = ordinary ring; 2 = upper tangent arc; 3 = parhelion; 4 = large ring; 5 = circumzenithal arc. The anthelion cannot be shown in this way of representation. (Cliché "Hemel en Dampkring").

Two things were particularly noteworthy in this halo: the strong development of the large ring and the presence of the anthelion in the absence of a parhelic ring. It should further be pointed out that of the other phenomena the upper tangent arc was the most intense, the small ring remaining faint.

B. The explanation of the large ring has only recently been made a point of closer investigation. After it had already been recognised as the equivalent of the ordinary ring for optical prisms of  $90^\circ$  by CAVENDISH (cf. 5, p. 79), little attention was further devoted to it, since the prisms in question were not to have any definite orientation. In my preceding publication I, accordingly, considered the large ring only from this point of view.

HASTINGS (11, p. 326), however, has pointed out that the possibility exists of an optical phenomenon which, whilst practically not to be distinguished from the large ring, yet arises in a different way, viz. in ice-prisms which lie almost or entirely horizontally. His explanation is very closely allied to that which BRAVAIS (5, p. 121, compare also 2, p. 72 and 13, p. 359; it is greatly to be regretted that EXNER in the second edition of PERNTER's handbook has omitted BRAVAIS' explanation) has given for the lateral tangent arcs to the large ring. HASTINGS points out that prisms in a horizontal plane have already refracting edges of  $90^\circ$  in all possible directions, so that with comparatively small

<sup>1)</sup> See also "Zur Erklärung der komplizierten Halos", published afterwards in the Meteorologische Zeitschrift, Nov. 1926, p. 411.



deviations from the strictly horizontal position of the principal axis, many of the optical prisms can already get a position of minimum deviation.

BRAVAIS' theory of the lateral tangent arcs to the large ring is in conformity with his explanation of the circumzenithal arc: to the latter all the prisms with an upturned basal surface contribute, thus giving a circle with the zenith as its centre. In the same way all the horizontal prisms with basal surfaces turned in one direction give a circle with its centre in this direction. There are, however, in the horizontal plane an infinite number of directions in which the prisms may be directed. Of the infinite number of circles which would theoretically be formed in this way, a certain number lie so close to each other that together and under favourable circumstances they produce a visible arc. Thus the lower lateral tangent arcs are formed by rays of light entering through a basal surface and leaving the crystal through a side face, and the upper lateral tangent arcs (sun's altitude below  $30^\circ$ ) when the rays of light pass the faces in inverse order. These latter must turn their convex side towards the sun, and for this very reason already they coincide mainly with the large ring. In BRAVAIS' time they had not yet been observed. In his dissertation BESSON (2, p. 72) mentions two cases observed by himself and his colleague DUTHEIL. Afterwards he has recorded another very positive observation of these arcs (3, p. 5). I myself had for a long time already, considered the possibility that a number of cases, in which mention was made of the large ring, would really refer to the arcs in question. In my publication of 1919 (16, p. 31) I already alluded to this with reference to the halo of Dec. 29, 1914 (Fig. 11; taken from 8). In a publication in "Hemel en

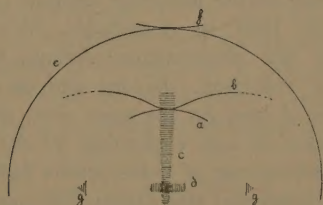


Fig. 11. Halo of Dec. 29, 1914.

*a* = ordinary ring; *b* = upper tangent arc; *c* = light column; *d* = parhelic ring; *e* = large ring; *f* = circumzenithal arc; *g* = parhelion. (The parhelia and the ordinary ring were only seen by some of the observers). (Cliché K. N. M. I.)

Dampkring" (15, p. 37) I ascribed the large ring on Jan. 8 1919 (cf. 10, p. 51) to intensification caused by the upper lateral tangent arc. On Febr. 21, 1918 (cf. 9, p. 155), and on March 28, 1920 (cf. 18, p. 55) intensified portions of the large ring were clearly observed by me; in the latter case the height of the most intense part was estimated, and found to correspond to the height required by BRAVAIS'

theory. In most of these cases (with the exception of Jan. 8, 1919) the upper tangent arc to the ordinary ring was particularly strongly developed, both as regards intensity and extension. This suggests the great prevalence of *horizontal ice rods*, which, as said before, would have to be held responsible for the upper (and lower) lateral tangent arcs to the large ring.

Hence the conclusion should be drawn in my opinion that in a number of cases in which (with solar altitudes  $< 20^\circ$ ) the large ring was observed in complex halos — accompanied by an intense tangent arc to the ordinary ring, this ring itself being faint — this large ring was caused by almost horizontal ice rods, and has to be explained by the theory of BRAVAIS for the upper lateral tangent arcs, or by that of HASTINGS. I cannot, however, agree with HASTINGS that this explanation of the large ring can replace the earlier one. The observations of the large ring at the same time with an intensive ordinary ring which I published before (16, p. 31, 36) can only be explained in the old way.

These considerations may elucidate to some extent, how, given the practical impossibility of seeing the large ring detached from the tangent arcs under consideration, these tangent arcs, which were predicted by the ingenious BRAVAIS, could remain considered as non-existent for so long a time.

Now recurring to the halo of Dec. 23 1925, we arrive at the conclusion that both the very intensive and extensive upper tangent arc to the ordinary ring and the intensive large ring point to a particular development of the horizontal ice rods.

C. Besides the striking brightness of the large ring and upper tangent arc, it was the presence of the anthelion *without parhelic ring* that made the halo of Dec. 23, 1925 so remarkable. We might also give the following account of this phenomenon: Among the infinite number of possible directions of reflection, the one with an angular difference of  $180^\circ$  was the privileged one. As we have seen, according to BESSON's theory there is question of such a privilege of the direction of  $180^\circ$  in the occurrence of combinations of 4 ice prisms fused together with their points and falling with vertical side faces: The "double mirrors" can be turned over a considerable angle in the horizontal plane, and all the same reflect the light from the direction of the anthelion. Every other reflecting vertical face will at the turning, send out the light every time in another direction.

These aggregates may, further, be the crystals that fall most undisturbedly in horizontal position: The air having an opportunity of escaping between the arms, the "Schaukeln" to which the ice plates are liable, need not occur here. By this circumstance it is rendered possible that the anthelion appears exactly at the sun's altitude.

If we now accept BESSON's theory, we must assume that with the

halos under consideration the particular circumstance presents itself, that the ice prisms have combined in large numbers to + shaped aggregates. When the circumstances of temperature etc. in that cloud had to give rise to the occurrence of aggregates, the chance was still greater that also the combination of 2 would occur, i.e. the so called holohedric prisms. That these were actually present in great numbers, is rendered probable by the strong development of upper tangent arc and large ring; in consequence of the greater stability in the horizontal orientation which the holohedric prisms show compared with the hemimorphous ones, the more common phenomenon (in casu the upper tangent arc) becomes exceedingly intense and extensive, the necessarily less intense and consequently rarer phenomenon (in this case the upper lateral tangent arcs to the large ring) coming in the region of visibility. Another factor that can contribute to the greater intensity is the double length of the holohedric prisms. If once such a crystal has the right position to give a definite point of the halo, twice as much light passes as a hemimorphous prism would have given in this position.

#### 4. The Very Complex Halos.

A. It seems to me that from the above a far-reaching conclusion may be drawn with reference to the explanation of complex halos:

*Under rare circumstances the hemimorphous ice prisms — which already under ordinary circumstances are able to produce several halo-phenomena at the same time (cf. p. 173) — begin to join to combinations of 2, 3, and 4. If this is the case "the" rare halo-phenomena may be added to a halo. These are: the upper and lower lateral tangent arcs to the large ring, the paranthelia and the anthelion.*

B. Besides in the observation of Dec. 23, 1925 and the other halos already mentioned, this hypothesis finds a firm support in the statistical data published by MEYER in 1925 (12, p. 13, cf. also 21, p. 191).

The part of the "Table of Relationship" referring to the 3 rare phenomena in question, is reprinted here. Every value has been calculated

by MEYER according to the formula  $\frac{G^2}{N_1 N_2}$ , in which  $G$  is the number

	Ordinary ring	Large ring	Upper and lower tangent arcs to the large ring	Lower lateral tangent arcs to the large ring	Parthelia	Circumzenithal arc	Light column	Parhelic ring	Paranthelia	Anthelion	Arcs through anthelion
Lower lateral tangent arcs to the large ring	1	2	2	—	3	0	0	7	24	8	13
Paranthelia	3	4	3	24	4	2	0	14	—	4	4
Anthelion	1	4	2	8	2	1	0	9	4	—	40



of observations in which the two phenomena the relationship of which is to be determined, occurred simultaneously, and  $N_1$  and  $N_2$  the total number of observations of each separately. The values thus obtained have been multiplied by 100.

Very recently in his discussion of MEYER's work in the *Meteor. Zeitschrift* (21) WEGENER pointed out that the anthelion according to MEYER's table shows close relationship to the lateral tangent arcs to the large ring. To this may be added that it is also much more closely allied to the large ring itself than to the ordinary ring. In the light of our hypothesis these relationships become very comprehensible. The relation to the large ring suggests that, among the circles observed simultaneously with the anthelion, there will probably be some not well distinguished upper lateral tangent arcs. That also the paranthelia (3-aggregates) present a very close relationship to the lower lateral tangent arcs is a surprising result. In contrast with WEGENER, who questions the validity of the theory of the paranthelia — because they are so closely allied to the halo-phenomena caused by refracting edges of  $90^\circ$  — I arrive at the conclusion that the theory (but then that of BESSON) is certainly valid. The relationship is, however, not to be looked for in the angles of the crystals, *but in the appearance of the phenomenon of "aggregation"*.

And this again makes it clear how the study of the halo-phenomena might lead to conclusions about the state of the atmosphere.

*Postscript during the correction of the Dutch edition.* After this communication had been presented to the Academy there appeared a publication by WEGENER (22), in which inter alia, a new theory is given of the arcs through the anthelion not discussed here by me. The anthelion itself is, then, considered as the luminous nodal point of the arcs. Though WEGENER's theory seems plausible for the explanation of the arcs in question, it cannot be accepted as the only correct one for the anthelion. This appears already directly from the fact that the anthelion of Dec. 23 1925 was observed with a solar altitude  $< 14^\circ$  (i.e. between  $12^\circ 15'$  (10.40 a.m.) and  $13^\circ 21'$  (11.07 a.m.)), whereas the lowest limit of visibility according to WEGENER lies at a solar altitude of  $14^\circ.1$ . Hence the explanation of BESSON, ignored by WEGENER, will have to be maintained at least by the side of the new theory.

## LITERATURE.

1. BESSON, L. (1907). Nouvelle Théorie de l'Anthélie, des Paranthélies, etc. *Comptes rendus de l'Acad. des Sciences. Paris*, CXLIV, p. 1190.
2. BESSON, L. (1909). Sur la Théorie des Halos. Dissertation. Paris.
3. BESSON, L. (1913). Notices sur divers phénomènes optiques observés à Paris. *Annales de l'Observatoire de Montsouris*. XI.
4. BESSON, L. (1923). L'Anthélie, les Paranthélies et les Halos blancs. *L'Astronomie*, XXXVII, p. 371.
5. BRAVAIS, A. (1847). Mémoire sur les Halos. *Journal de l'Ecole polytechnique Paris*, 31e Cahier.
6. DOBROWOLSKI, A. (1903). La Neige et le Givre. *Resultats du Voyage du S. Y. Belgica (Météorologie) Anvers*.

7. DOBROWOLSKI, A. (1916). Les cristaux de glace. Arkiv for kemi mineralogi och geologi, VI, 7. Stockholm (cited after BESSON).
  8. EVERDINGEN, E. VAN. (1915). De halo van 29 Dec. 1914. Hemel en Dampkring 12, p. 130 (cf. also: Onweders, opt. verschijnselen enz. in Nederland in 1914, p. 113).
  9. EVERDINGEN, E. VAN. (1918). Een feestdag voor Halowaarnemers (21 Febr. 1918), H. en D. 15, p. 155 (cf. also: Onweders, opt. verschijnselen enz. in Nederland in 1918, p. 59).
  10. EVERDINGEN, E. VAN. (1921). Onweders, optische verschijnselen enz. in Nederland in 1919 (Halo van 8 Jan. 1919, p. 51, halo van 24 Maart 1919, p. 55).
  11. HASTINGS, CH. S. (1920). A General Theory of Halos. Monthly Weather Review, June 1920, p. 322—330.
  12. MEYER, R. (1925). Haloerscheinungen. Abh. des Herder-Institutes zu Riga. Band I, no. 5.
  13. PERNTER, J. M. (1906). Meteorologische Optik. 3. Abschnitt. Wien und Leipzig.
  14. PERNTER-EXNER. (1922). Meteorologische Optik. 2. Auflage. Wien und Leipzig.
  15. PINKHOF, M. (1919a). Naar aanleiding van de halo's van 8 Jan. en 24 Maart 1919. Hemel en Dampkring 17, p. 36.
  16. PINKHOF, M. (1919b). Bijdrage tot de theorie der haloverschijnselen. Verhandelingen der Kon. Ak. v. Wetensch. te Amsterdam. Eerste Sectie, deel XIII, No. 1.
  17. PINKHOF, M. (1920a). Beiträge zur Halotheorie. Meteor. Zeitschr. XXXVII. Heft 3/4, p. 60.
  18. PINKHOF, M. (1920b). De halo's van Maart 1920. Hemel en Dampkring, 18, p. 43 en 53. (Cf. also: Onweders, optische verschijnselen enz. in Nederland in 1920, p. 58).
  19. PINKHOF, M. (1926). De Halo van 23 Dec. 1925. Hemel en Dampkring 24, p. 86.
  20. WEGENER, A. (1915). Verschwisterte und vergesellschaftete Halos. Meteor. Zeitschrift XXXII, Heft 12, p. 550.
  21. WEGENER, A. (1926a). Besprechung von R. MEYER, Haloerscheinungen. Meteor. Zeitschrift XLIII, Heft 5, p. 190.
  22. WEGENER, A. (1926b). Theorie der Haupthalos. Aus dem Archiv der deutschen Seewarte XLIII, N<sup>o</sup>. 2, Hamburg.
-

**Physiology.** — *Radiation-substances and Cardiac Hormones.* By Prof. H. ZWAARDEMAKER.

(Communicated at the meeting of November 27, 1926.)

The potassium-salt that is invariably present in the circulation fluid of animal organisms, is an indispensable constituent. For a number of systems, however, — I believe for all automatic systems — it can be replaced by any radio equivalent dose (1916<sup>1</sup>). It is particularly this radio equivalence (1 micro-erg per second, and per grm of heart-muscle) that in the last two or three years has been demonstrated by detailed ash-determinations<sup>2</sup>). The radio-active element (Potassium, Rubidium, Thorium, Uranium, Ionium, Radium, Radon) can also be substituted by corpuscular radiation, either alpha-, or beta. *Not*, however, by rays of light or Röntgen-rays, in the intensities used by us. In such experiments we started from isolated organs, from which  $\frac{1}{3}$  of the original potassium had been washed out<sup>3</sup>). The fittest object is the heart of cold-blooded animals, which stops beating as soon as the removal of the potassium has proceeded so far. The standstill is definite, provided the perfusion with potassium-free Ringer's solution be continued uninterruptedly. The beta-rays of mesothorium or radium<sup>4</sup>), the alpha-rays of polonium<sup>5</sup>), likewise cathode-rays<sup>6</sup>), evoke in 90 % of the hearts thus radiated, new regular pulsations that can persist for hours<sup>7</sup>).

In all these cases, a threshold-value is found.

There is also a latent period. We were surprised to see that in cases of feeble radioactivity these latencies sometimes last for hours. The latter circumstance leads us to suppose that there is some indirect causal connection between the new radioactivity and recovery. It is reasonable to

<sup>1</sup>) H. ZWAARDEMAKER, Verslag Kon. Akad. v. Wetensch. Amsterdam, 30 Sept. 1916; Proc. Vol. 19, p. 633; PFLÜGER's Arch. Bd. 173, p. 28, 1918; Ergebnisse der Physiol. Bd. 19, p. 326, 1921; PFLÜGER's Arch. Bd. 205, p. 20, 1924; Q. J. of exp. Physiol. Vol. 14, p. 338, 1924, etc.

<sup>2</sup>) G. M. STREEF, Thesis. Utrecht 1926.

<sup>3</sup>) A. J. CLARK J. of Pharm. a. exp. Therapeut., Vol. 18, p. 423, 1922; H. ZEEHUISEN, Ned. Tijdsch. v. Geneesk. 1926, I, p. 2123.

<sup>4</sup>) H. ZWAARDEMAKER, C. E. BENJAMINS, T. P. FEENSTRA, Ned. Tijd. v. Geneesk. 1916, II, p. 1923; H. ZWAARDEMAKER, Verslag Kon. Akad. v. Wetensch. Amsterdam, 31 Maart 1917; Proc. Vol. 19, p. 1161.

<sup>5</sup>) H. ZWAARDEMAKER en G. GRIJNS, Arch. néerl. de Physiol., t. 2, p. 500, 1918; Stralenterapie Bd. 21, p. 215, 1925; PFLÜGER's Arch. Bd. 213, p. 757, 1926.

<sup>6</sup>) H. ZWAARDEMAKER en T. P. FEENSTRA, Verslag Kon. Akad. v. Wetensch. Amsterdam, 27, juni 1925; Proc. Vol. 28, p. 650. (or weak X-rays?)

<sup>7</sup>) H. ZWAARDEMAKER, PFLÜGER'S Arch. Bd. 213, p. 763n. 1926.



assume that stimulating chemical substances are formed *by the radiation*, which substances might be called "automatins", (a term introduced by BENJAMINS already a twelvemonth ago), and which in their turn furnish the automatically moving organ with the so-called primary stimulus. Our "automatins" are purely hypothetical substances, they have never been seen in fact. Moreover, the existence of such substances was suggested by recent researches<sup>1)</sup>, demonstrating the formation of so-called "ions", or carriers, also in liquids. The "ions" generated by the radiation in the tissue (some of which reconstruct the complete molecules from which they have originated, while others remain separate) are, as we believe, our hypothetical "automatins", or give them origin.

If we assume the automatins to be dialysable, they will be regularly washed out of the beating heart, whose lacunae are emptied out at every contraction, and replenished at every diastole. When, with potassium or with its substitutes, their formation remains too weak, it is easy to find an explanation for the groupings and the brief standstills. But it is not so easy to explain the very late after-effects we are confronted with in half the radiation-experiments. They almost make us believe in the cooperation of an autocatalysis. We will picture a case in which the pulsations were resumed in consequence of the action of cathode-rays (or weak X-rays) (Fig. 1). The later group lasted here about 24 hours (Fig. 2).

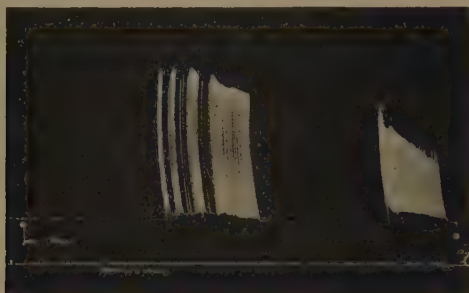


Fig. 1.

Of course, after the organ had ceased to contract through deprival of potassium we waited some time. There is a definite correlation between such a standstill, before starting the radiation and the latent period between the commencement of the radiation and the moment of revival. Those periods generally keep pace with each other. Thus the times before and after the beginning of the radiation present a sort of symmetry, with weak, though sufficient radiation (1.6 mgm Ra in 1 cm<sup>2</sup> of enamel) the latency

<sup>1)</sup> A. KAILAN, Liebig's Ann. Bd. 433, p. 272, 1923.

is somewhat longer than the time of waiting, whereas with somewhat intenser radiation (1.6 mgm Ra + 3.3 mgm from a distance of 0.5 cm)



Fig. 2.

the periods are equal. This (it should be understood) is an average result of a large number of experiments, for the varieties are many.

The duration of the latent period depends not only on the time of waiting, but also on the dosis of the radiation. When, in former times we fixed it at f.i. 2 mgm-hours, the latent period amounted to only a few minutes, half-an-hour, or an hour; now that we have been working of late with threshold values of f.i. 1 mgm-hour, the revival will often manifest itself after hours.

The connection between the interval of waiting and latency, as well as that between the intensity of radiation and latency, lends support to the probability of existing intermediate chemical substances, which first have to accumulate and to penetrate as far as the receptors in the nodal tissue, before contractions can be evoked in the sinus or the atrioventricular node, which can propagate over the whole heart-muscle.

Our hypothesis that automatins originate *through radiation*, would have remained a dead letter, because the substances are not perceptible, if not new, unexpected investigations by others had offered a wider field of inquiry.

It so happened, namely, that the same conception was brought forward from two entirely different quarters.

L. HABERLANDT of Innsbruck has published a series of researches on a hormone of the cardiac movement <sup>1)</sup>, showing that in the sinus and in the atrioventricular node of the frog's heart there is a substance, extractable by means of Ringer's solution resp. absolute alcohol, which when administered to a ventricle which is badly beating, or standing still, heightens an existing frequency, improves a faulty rhythm, and restores the cardiac action. This substance is dialysable, insoluble in ether, and resistant to a temperature of 100°.

<sup>1)</sup> L. HABERLANDT. *Ergeb. der Physiol.* Bd. 25, p. 86, 1926.

At the same time J. DEMOOR <sup>1)</sup> of Brussels made experiments, with which from the nodal tissue of mammalia, aqueous or alcoholic extracts were obtained, which, when administered to the isolated left auricle of the rabbit's heart, made it beat rhythmically and regularly, which does not happen without administering these extracts. The active substances, concerned here, are dialysable, they do not resist a temperature higher than 70° C., and are destroyed in a diluted solution within 24 hours.

Do these researches bear any relation to mine? Prof. NOYONS broached this subject and thereby gave a welcome impetus to the studies that occupy us. Moreover, Prof. HABERLANDT kindly imparted to me his view of the nature of his substances. He also suspected the possibility of a relation between our results. A possibility or a probability?

A very simple experimentum crucis appeared, by which this question might be set at rest. At once I availed myself of it.

A frog's heart on Kronecker's cannula is brought to a standstill through deprival of potassium. After a quarter of an hour I placed 1.6 mgm of Ra in 1 cm<sup>2</sup> enamel close to the ventricle, which is exposed to full radiation from that moment. After a latent period of half an hour the pulsations recommence.

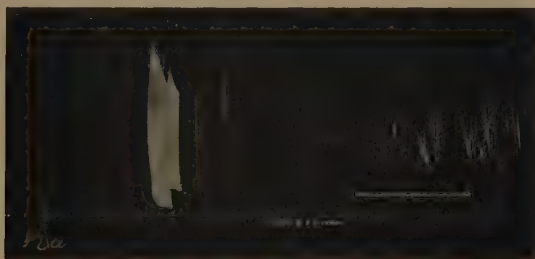


Fig. 3.

During the pulsations, thus produced, we collect the out-flowing liquid, after having diminished the rate of perfusion (without interrupting it) as much as possible (underlined in the figure). With this liquid we perform HABERLANDT's experiment on another ventricle on Straub's cannula. The ventricle that comes to a standstill on potassium-free Ringer's mixture, commences with grouped pulsations, and soon attains a regular-rhythm of about half an hour's duration.

We were thus able to perform quite a series of experiments. Not one of them turned out negative, while in control-tests a Straub-heart, arrested by deprival of potassium did not resume its beats on repeated renewal of its contents with ordinary potassium-free liquid.

<sup>1)</sup> J. DEMOOR, Arch. Int. de physiol., 20—23; résumé in: les transmissions humorales, Bruxelles médical, 13 Sept. 1925.



Besides by radiation with the beta-rays of Radium we have revived hearts also by alpha-rays of Polonium (from three sides, for the penetrating power

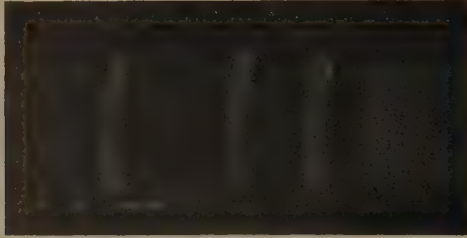


Fig. 4.

of polonium-rays is low (in water 32 microns). Well then, also the potassium-free Ringer's solution, that passes slowly through such a Polonium-heart, makes a heart which has ceased to contract (on a Straub-cannula) recover its beats in a short time (a few minutes) <sup>1)</sup>.

It is a fact, then, that automatins are formed as well by alpha-, as by beta-rays. Are they homogeneous, or antagonistic, as the physiological radio-antagonism leads us to suspect? <sup>2)</sup> This problem is being investigated. I may be allowed to confine myself for the present to the hypothesis that, on the basis of our actual experience, the beta-radiation substances are to be considered identical with HABERLANDT's and DEMOOR's hormones.

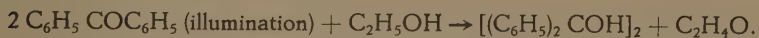
<sup>1)</sup> Future critics of experiments like those discussed in this paper, should remember that the revival through automatins after repeated renewal of the contents, takes place with a latency of a few minutes. Recovery after half an hour, without renewed filling, would not, of itself, prove anything, as this is not surprising in a Straub-heart, left to itself, in which sooner or later potassium is always set free,

<sup>2)</sup> I refer to my contribution to GRIESBACH'S "Jubilee volume" Giessen, 1925, p. 135.

**Chemistry.** — *The Light Oxydation of Alcohols as Contribution to the Knowledge of the Photo-Chemical Phenomena.* By J. BÖESEKEN and S. L. LANGEDIJK.

(Communicated at the meeting of December 18, 1926).

§ 1. The researches made by one of us for some ten years with collaboration of W. D. COHEN<sup>1)</sup> and H. D. MULLER<sup>2)</sup>, had yielded the result that there are a number of aromatic mono- and aliphatic  $\alpha$ -diketones which are sensitive to light in the extreme part of the visible spectrum. Exposed to the light they can oxidize a number of hydrogen compounds, especially the primary and secondary alcohols, to the corresponding aldehydes and ketones, in which they themselves are reduced to pinakones, e.g.:

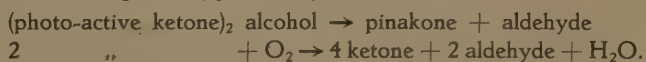


If there is sufficient oxygen present, an oxidation takes place of the alcohol to the aldehyde (ketone), in which the aromatic ketone remains seemingly unchanged, thus playing the part of photo-catalyst.

It then appeared that the quantities of pinakone in the first case and the consumed oxygen in the second case were equivalent, when for the rest the circumstances were the same, i. e. that two mol. of pinakone were formed in the same time that one mol. of oxygen was consumed.

If further appeared that already at moderate concentration the velocity of reaction became little dependent on the concentration of the ketone, that it was proportional to the concentration of the alcohol, and — when the circumstances were chosen the same for the rest — proportional to the *square* of the light intensity. With a view to this a scheme of reaction was drawn up by one of us, in which it was assumed that the velocity of formation was *measured* of a compound from two activated benzophenon-molecules with one mol. of alcohol:

2 Photo-active ketone + 1 alcohol = (photo-active ketone)<sub>2</sub> alcohol, which compound could then react with much greater velocity in two directions according as oxygen was present or not:



In order to be able to give an idea of the course of conversion, we must give an account of the succession of the processes:

If a certain quantity of ketone is struck by the light, the intensity of

<sup>1)</sup> W. D. COHEN, Proefschrift, Delft, 1915.

<sup>2)</sup> BÖESEKEN, Recueil. Trav. Chim., **40**, 433 (1921).

the light will be unequally distributed in consequence of the absorption of the light in the vessel, and it will diminish from  $I_0$  to  $I_0 e^{-\kappa c x}$ , dependent on the absorption coefficient of the ketone ( $\kappa$ ) and on its concentration ( $c$ ) over the cross-section of the vessel ( $x$ ). Accordingly at the moment of the illumination there will be a number of active molecules, the concentration of which decreases from the light side to the dark side according to this law.

Now the question may be raised, what happens immediately after this activation? If these molecules act as perfect mirrors, i.e. if they send back the light energy with the same velocity, we could only speak of reflecting or scattering, and the concentration of the active molecules would remain insignificant. The fact that in the absence of appreciable reflection and scattering a vigorous absorption is observed, suggests that the activated molecules have time to transfer their activity to themselves or to other molecules.

A. If we assume in the first place that this time is very short, they will only be able to bring about this transference at the place itself or very close to the place of their activation. The concentration in an arbitrary layer of light in active benzophenone molecules may then be calculated as follows. By the illumination a number of active ketone molecules is formed proportional to the quantity of light present there. If  $S$  is the illuminated surface, the quantity is:

$$= K_1 S I_0 e^{-\kappa c x} \cdot \kappa c dx.$$

Part of these will be converted; if we assume this to be a monomolecular reaction (that we may, therefore, disregard collisions of equal molecules) the number of disappearing molecules  $= K_2 C_{\text{active-ketone}} S dx$ .

A stationary state will result, when these two reactions are in equilibrium; from this ensues that in every layer of light the concentration of the active ketone mol.  $= K_1 I_0 e^{-\kappa c x} \kappa c$ . These active ketone molecules now get secondarily in contact with alcohol molecules, in which the ternary photoactive compound (see further) originates:

$$\frac{d[\text{tern. comp.}]}{dt} = K_2 C_{\text{act. ket.}} \cdot C_{\text{alcohol}} = K_2 K_1^2 I_0^2 e^{-2\kappa c x} \cdot \kappa^2 c^2 \cdot C_{\text{alcohol}}$$

From this the total quantity of this compound in the layer can be calculated by multiplication by  $S dx$ , and then by integration the total quantity in the vessel is:

$$\int_0^x S K_2 K_1^2 I_0^2 e^{-2\kappa c x} \kappa^2 c^2 C_{\text{alcohol}} \cdot dx = \frac{1}{2} S K_2 K_1^2 I_0^2 (1 - e^{-2\kappa c x}) \kappa c C_{\text{alcohol}}$$

$$\text{The concrete increase} = \frac{d[\text{comp.}]}{dt} = \frac{1}{2} \frac{K_2 K_1^2}{x} I_0^2 (1 - e^{-2\kappa c x}) \kappa c C_{\text{alcohol}}.$$

B. If we assume that the activated benzophenone molecules lose their activity comparatively slowly, so that in a thin layer and at not too



low temperature we may assume that they have time to spread evenly through the reaction space without having lost much of their activity, a stationary state will have established itself after a short space of time, which may be represented by:

$$\int_0^x K_1 S I_0 e^{-\kappa c x} \cdot \kappa c dx = K_1 I_0 (1 - e^{-\kappa c x}) S = K_2 c_{\text{active mol.}} \cdot x \cdot S.$$

or

$$c_{\text{act. mol.}} = \frac{K_1}{K_2 \cdot x} I_0 (1 - e^{-\kappa c x}).$$

If we, therefore, suppose a sufficient duration for the light-active molecules, and if we choose a small and equal thickness of the layer, since this is necessary for a sufficient absorption of oxygen, we may use for concentration of the active molecules the expression:  $\frac{K}{x} I_0 (1 - e^{-\kappa c x})$ , and express the velocity reaction by:

$$\frac{d [\text{active comp.}]}{dt} = \frac{K^2 I_0^2}{x^2} (1 - e^{-\kappa c x})^2 \cdot c_{\text{alcohol.}}$$

Whatever image we may use here, always the square of the light intensity appears; a choice between the two views can only be made by the determination of the reaction velocity under different circumstances.

§ 2. *The photo-active region.* It had already been rendered probable by COHEN that the photo-active region lies in the extreme visible violet and that the ultra violet, for so far as the region of the mercury lines occurring in it is concerned, was inactive.

On absorption of the visible light by a solution of nitroso dimethylaniline, no reaction could be observed of this light in quartz tubes. For the rest there remained much uncertainty about the extension of the photo-active region.

One of us has defined this active area more closely with H. D. MULLER in the following way. We made use of another reaction, which, however, likewise ensues from the formation of the ternary compound: (photo-ketone)<sub>2</sub> alcohol.

If an alcoholic solution of  $\text{Hg Cl}_2 + (\text{NH}_4)_2 \text{C}_2\text{O}_4$  is illuminated in the presence of the above-mentioned ketones,  $\text{Hg Cl}$  is deposited in the sunlight and in the light of the mercury lamp.

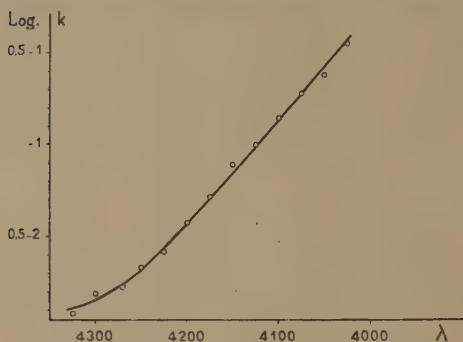
We then laid a series of capillary tubes filled with this solution, under the microscope, and illuminated them by means of a spectrum obtained by the aid of a strong PHILIPS microburner.

It could then clearly be observed that in the tube which was in the extreme violet between 400 and 410  $\mu\mu$ , the first  $\text{HgCl}$  crystals appeared;

not until some days later were there also seen crystals on either side of this tube, both to the side of the red and to that of the ultra-violet in a greatly decreasing degree. These crystals being of about the same size and the tubes having the same diameter, their number was an approximate measure of the converted quantity of substance. Though this number decreases exceedingly rapidly, we may yet conclude that the photo-active region is not confined to a few light vibrations.

Then the question suggested itself if this region of photo-activity with a clearly-marked top coincided with a region of general absorption.

As the Technical University is not fully equipped with apparatus for accurate determinations, one of us has called in the hospitality of



the physical laboratory at Utrecht, which is under the direction of Prof. ORNSTEIN. Through the very kind assistance met with there, he has succeeded in defining the whole area of absorption of some ketones both in the visible light and the ultra violet part of the spectrum adjoining it.

In this there was not found the slightest indication of an absorption top in the violet part. We are here in the ascending part of the region of absorption, but with the three investigated ketones we are very far from the top, which lies in the ultra violet part of the spectrum, where we have not met with any reaction. The fact, indeed, that the photo-active region is narrowly limited indicates that it is quite surrounded by the region of total absorption.

We see that part of the light energy, and undoubtedly only a very small part, can be applied for this oxidation reaction, hence that the photo-chemical efficiency is small. It is certain that when the light is thought divided in separate vibrations, there are some among these vibrations that possess a very high efficiency; the more we are, however, struck then with the slight efficiency of the light vibrations from the rest of the absorption region.

If the photo-active region were confined to very few light vibrations, we might imagine the case that the photo absorption top was so sharp that it was not observed in the determination of the total absorption region. The experiments with the capillaries show, however, that the photo active region in question is not to exceedingly narrow.

It is possible that one single ray of light possesses an efficiency of 100 %, and it would certainly be of great importance to refine the means of investigation to such an extent that this too could be found.

It follows from our investigations and those by COHEN that the tops of the photo-active regions for different ketones lie "in the neighbourhood" of the mercury lines in the violet, and that these regions certainly overlap for an important part, (see below). It is, however, a priori probable that the real top occupies a separate place in the spectrum for every ketone. For the rest the general absorption is very sensitive to admixtures; it was found by us that an alcoholic solution of the ketones yielded a spectrum which was turned more towards the red than a solution in petroleum ether.

When the question is now considered what happens with the rest of the absorbed energy, it is undoubtedly conceivable that *other chemical reactions* may be started by it, e.g.:  $\text{C}_6\text{H}_5\text{COC}_6\text{H}_5 + \text{C}_2\text{H}_5\text{OH} \rightarrow \text{C}_6\text{H}_5\text{COO}_2\text{H}_5 + \text{C}_6\text{H}_6$ , which then take place somewhere in the ultraviolet absorption region, etc. Under the circumstances chosen by us, the rest of this energy is probably entirely converted to heat.

With somewhat complicated molecules no theoretical light efficiency can in general be expected throughout the whole absorption region, nor is this, certainly, the case with the assimilation process. Here, too, the photochemical region is encompassed by the general absorption region of the chlorophyll. The maximum efficiency is not reached at the characteristic top in the red, (645–670  $\mu\mu$ ), it lies probably more in the yellow between 570–645  $\mu\mu$ , hence here (O. WARBURG and E. NEGELEIN Z. phys. Ch. 102, 235 (1922)) past the top.

§ 3. Some mixtures of ketones in alcoholic solution were illuminated by W. D. COHEN in order to examine whether disturbances appeared then, and if so what disturbances. Because with somewhat considerable concentration of the ketone the reaction velocity seemed to be independent of this concentration, it was at first expected that the two reactions would not interfere with each other. This appeared to be by no means the case.

If we consider the velocity of formation of the pinakones:

$$-\frac{d[\text{ternary comp.}]}{dt} = +\frac{d[\text{pinakone}]}{dt} = \left[ \frac{I_0 K}{x} (1 - e^{-\kappa cx}) \right]^2 c_{\text{alcohol}}$$

then with somewhat considerably concentration and with *perfect specific absorption* the increase of the concentration will really have little influence on the velocity of formation of each pinakone separately, as  $(1 - e^{-\kappa cx})$  rapidly approaches to the unit.

When they are mixed, however, and when the absorption regions coincide at the places of the photo-activity, the two ketones will divide this light according to their power of absorption and their concentration.

One ketone will partly be in the path of light of another ketone, so that we then might speak of an *interior filter action*. The law of this,



formulated by LUTHER and WEIGERT, Z. Phys. Ch. **53**, 408 (1905), has been applied by one of us (LANGEDIJK) as follows:

If a vessel with two substances in the same solvent with the concentrations  $c_A$  and  $c_B$  is exposed to monochromatic light, the absorption coefficients being  $\kappa_A$  and  $\kappa_B$ , the intensity of the light after having traversed a layer  $dx$  is diminished by the amount:

$$\begin{aligned} -dI &= (\kappa_A c_A + \kappa_B c_B) I dx \\ \frac{dI}{I} &= -(\kappa_A c_A + \kappa_B c_B) dx \\ \ln I &= -(\kappa_A c_A + \kappa_B c_B) x + C \\ I &= C' \cdot e^{-(\kappa_A c_A + \kappa_B c_B) x}. \end{aligned}$$

For  $x=0$   $I=I_0$  hence  $C'=I_0$ , from which follows:

$$I = I_0 e^{-(\kappa_A c_A + \kappa_B c_B) x}.$$

Then the total absorption, after having traversed a layer  $x$ , is then <sup>1)</sup>:

$$H = I_0 [1 - e^{-(\kappa_A c_A + \kappa_B c_B) x}].$$

If we further call  $H_A$  the quantity of light absorbed by A, and  $H_B$  that absorbed by B, the following equation is valid for a thin layer:

$$dH = dH_A + dH_B = -dI = (\kappa_A c_A + \kappa_B c_B) I \cdot dx$$

and we may think this distributed in this very thin layer as follows:

$$dH_A = \kappa_A c_A I dx \text{ and } dH_B = \kappa_B c_B I dx.$$

$$\text{Now } I dx = - \frac{dI}{\kappa_A c_A + \kappa_B c_B}$$

$$dH_A = - \frac{\kappa_A c_A}{\kappa_A c_A + \kappa_B c_B} dI \text{ and } dH_B = - \frac{\kappa_B c_B}{\kappa_A c_A + \kappa_B c_B} dI$$

If these values are integrated, we get as above:

$$H_A = \frac{\kappa_A c_A}{\kappa_A c_A + \kappa_B c_B} I_0 [1 - e^{-(\kappa_A c_A + \kappa_B c_B) x}]$$

$$H_B = \frac{\kappa_B c_B}{\kappa_A c_A + \kappa_B c_B} I_0 [1 - e^{-(\kappa_A c_A + \kappa_B c_B) x}].$$

If, therefore, there are some substances present in a solution, they divide the absorbed light of a definite wave-length — hence also the photo-active light — in proportion to the products of their concentrations and absorption coefficients.

Since with the reaction velocities of our ketone reductions the squares of this value come into play, it is clear why COHEN observed such strong

<sup>1)</sup> This derivation of the total absorption in the presence of two (and more) substances, has already been given by PESKOW, Journ. Russ. Chem. Ges. **47**, 918(1915), Journ. Phys. Chem. **21**, 382 (1917).



20 cc.) and illuminated for 30 hours in a light-thermostat with 30 volts and 3.5 ampères — the oxygen absorption being 0.012 cc. per minute. The alcohol distilled off exhibited *no rotation*. The experiment was repeated with methyl-ethyl carbinol. There were absorbed 0.14 cc.  $O_2$  per minute. The carefully fractionated methyl-ethyl carbinol showed faint levo-rotation at 60 volts and 4.1 Ampères after 100 hours' illumination, in which, therefore,  $6 \times 0.14$  litres  $O_2 = 0.84$  l had been absorbed, equivalent to 2.5 grammes of alcohol. In a 2 dm. tube this amounted to  $-0^\circ.04$ ; a repeated experiment gave  $-0^\circ.03$ . Though the amount is very small, it is certainly above the error of observation, and we may say with pretty great certainty that in principle a decomposition has been effected by this way.

New points of views are opened in so far that it must be possible that in nature in the presence of compounds at the same time sensitive to light and optically active, new optically active compounds are built up by photo-chemical way.

In the assimilation process the formation of optically active glucose and of amyllum is not necessarily connected with the light process. This may be restricted to the activation of the chlorophyllcarbonate to a product of high potential energy, that now further under the influence of enzymatic dark reaction can change into oxygen and into optically active compounds.

Delft, Dec. 1926.

---



**Botany.** — On the favorable effect of 35° C. on the cell-formation in foliage-leaves of *Hyacinthus orientalis*. (Communication No. 22. Laboratory for Plant-physiological Research, Wageningen.) By Miss IDA LUYTEN. (Communicated by Prof. A. H. BLAAUW.)

(Communicated at the meeting of December 18, 1926).

### § 1. Introduction and Method.

In the research on the optimal temperature for the flower- and leaf-formation of *Hyacinthus Orientalis* (BLAAUW 1924) stress had already been laid on the fact, that the optimum for these two differs greatly. It appeared that if on shooting in spring one would be sure of the best flower-cluster the hyacinth-bulbs should after being lifted first be exposed to 26° C. for 8 weeks, next to 17° C. for 4½ weeks.

If however after being lifted the bulbs were exposed to 35° C. for 5 weeks and next to 17° C. (i.e. 17° for 7½ weeks till planting-time), it appeared that in spring the foliage was several cms. longer than in the other still favorable high temperature-exposures. The following question therefore was obvious: is this length of the foliage-leaves merely a result of greater stretching of the cells or have there more cells been formed on exposure to 35° for 5 weeks, so that the leaves grew longer, with equally long cells.

In order to trace this, cells must be counted in the longitudinal direction. The material for it was obtained as follows. We started with 140 bulbs "Queen of the Blues", circumference 80—90 mms, 2 years old, July 4, 1924. On this date at the beginning of the experiment, 60 bulbs were exposed to 35° C., and 60 bulbs to 17° C., and 20 bulbs were fixed in alcohol 96 %. After 5 weeks (Aug. 9, 1924) 20 bulbs were fixed from the two temperatures and the bulbs from 35° transferred to 17°. On Sept. 30, i.e. 12½ weeks from the beginning of the experiment, 20 specimens were fixed of the group exposed to 35° for 5 weeks, next to 17° for 7½ weeks. Likewise of the group constantly exposed to 17°. On this latter date the remaining bulbs (20 of each exposure) were planted; Febr. 11, 1925 they were lifted and fixed.

We subjoin a survey of the fixations.

July 4, 1924 Beginning of the experiment	Aug. 9, 1924 (after 5 weeks)	Sept. 30, 1924 (after 12½ weeks)	Febr. 11, 1925 (planted in the open on Oct. 1)
20 bulbs	20 bulbs from 35° 20 bulbs from 17°	20 bulbs from 35°—17° 20 bulbs from 17°	20 bulbs from 35°—17° 20 bulbs from 17°

Next each group of each date was examined in the following way.

The bulbs were put in alcohol 50 %, so that the tissues grew soft and supple, next they were peeled to the outermost foliage-leaf. On the data July 4, and Aug. 9 it was sometimes hard to see whether the outermost foliage-leaf had already been reached. For by that time there is but little difference outwardly between the last sheath-leaf and the first foliage-leaf. In doubtful cases a reagent may be used, consisting in the application of a solution of iodine and iodide of potassium to slight cuts in the tissue. In the sheath-leaf the incisions are stained blue on account of the starch they contain. In the foliage-leaf the colour is browner and disappears sooner. Under the simple dissecting stage-microscope of Zeiss the lengths of the outer foliage-leaves of the 20 bulbs were accurately measured and the average computed, except with the foliage-leaves on Febr. 11, that were already long. The 3 bulbs containing the foliage-leaf, the length of which approaches this average as near as possible, are kept apart. In these 3 bulbs the number of cells in the longitudinal direction of the outer foliage-leaves will be counted. For this purpose the entire terminal bud is cut from the bulb, and good care is taken, that besides the terminal bud some disc-tissue is removed, so that the outer foliage-leaf is not injured at the base.

Next the outermost foliage-leaf is removed and measured again. It frequently appears, that the leaf has grown slightly shorter in most cases sometimes a little longer; next sections are made by hand through the central-longitudinal axis of this foliage-leaf, as was done by Miss VERSLUYS (1925) for the foliage-leaves of the Hyacinth. This central part of the foliage-leaves of July 4 and August 9, 1924 could be cut in its entirety; on the later dates the leaves were too long and they were divided into pieces of ca 1 cm.; the 2 lowest pieces at the base were taken smaller, because they contained a great many cells. The series of longitudinal sections were made perpendicular to the plane of the epidermis, the leaf lying between elder-pith soaked in glycerine. The cells were counted at the backside, i.e. morphologically the undersurface, viz. the third row under the epidermis. When a vascular bundle has been struck, the backside may usually be recognised by the fact the phloem is always turned towards the backside of the leaf. The presence of a vascular bundle in the section being far from frequent, before cutting I always removed a piece of the epidermis with a sharp razor at the inside in the direction of the base, so that from this injury I could determine the backside and underside of that piece of the leaf. Before being stained the sections were tapped with the sides of a pincette; this was done in order to remove the starch from the lower parts of the leaves and besides to make the cell-walls extend normally, for these have been compressed too much in consequence of being jammed between the pieces of elder-pith. Next a 2 % solution of methylene-green is used for staining, so that the cellwalls are clearly visible. For the counting

a Zeiss microscope is used, provided with an objective A and a micrometer-ocular 2 of LEITZ. The entire scale of the micrometer (10 spaces) corresponds with 1.73 mm. of the preparation. Accordingly it may be counted, how many cells occur in 10 successive spaces.

§ 2. *On the length of the buds (outer foliage-leaves).*

Let us now first consider the results we get, when from each group on each date we measure the buds down to the base and compute the average. It was already mentioned above, that groups of 20 bulbs were fixed. Yet on a few dates the average has been taken from 18 and 19 bulbs, as a few bulbs could not be used, because either the apex of the outer foliage-leaf was injured or because the bulb contained two terminal buds instead of one. For the sake of accuracy this latter deviation was not included in the figures.

On the date of starting (July 4) no average was computed, though 20 bulbs were fixed, but three of an average length were selected by the eye. This group of bulbs had also to serve for a root-examination. If one wants to measure the bulbs accurately, the surrounding tissue should be cut away deeply, and consequently on account of the injury to the disc, the bulbs would have been of no value for the other research. On this date no average is given, but the lengths of all of the three buds measured.

Now it appears from table 1, that on August 9, 1924 there is not much

TABLE 1. Length of the terminal bud in mms.

July 4, '24 beginning of the ex- periments	Aug. 9, 1924 after 5 weeks		Sept. 30, 1924 after 12½ weeks		Febr. 11, 1925 results after 30½ weeks	
	after 5 weeks 35°	after 5 weeks 17°	5 w. 35° + 7½ w. 17°	12½ w. 17°	5 w. 35° + 7½ w. 17°	12½ w. 17°
3.3	<i>n</i> = 19	<i>n</i> = 18	<i>n</i> = 20	<i>n</i> = 20	<i>n</i> = 20	<i>n</i> = 20
3.2	3.11 ± 0.08	3.45 ± 0.13	23.13 ± 0.28	11.88 ± 0.41	180.3 ± 0.35	84.6 ± 0.20
3.3						

difference in length in the bulbs which were exposed either to 35° for 5 weeks or to 17° for 5 weeks; it might only be observed, that the length of the terminal bud in 17° is slightly increased, whilst the buds had not grown in 35° after this period.

September 30 however yields quite a different picture. After *both* groups have been exposed to 17° for another 7½ weeks, we see that the length of the outer foliage-leaf, previously exposed to 35° for 5 weeks, is *nearly twice as long* as in the bulbs constantly exposed to 17°.

On Febr. 11, 1925 the outer foliage-leaves, exposed to 5 weeks  $35^{\circ} + 7\frac{1}{2}$  weeks  $17^{\circ}$  are also more than twice as long as those exposed to  $12\frac{1}{2}$  weeks  $17^{\circ}$ . It follows that *the high temperature has a long after-effect and influence on the length of the foliage-leaves*. BLAAUW (1924) already pointed this out. The results I get from these macroscopic external measurements of these foliage-leaves, tally with the figures BLAAUW (1924, p. 16) found for the foliage-leaves of the same variety. For comparison we subjoin them, as far as observed at the time.

TABLE 2. Length of the outermost foliage-leaf in mm.

July 7 1922 Beginning of the expe- riments	Aug. 11, 1922 after 5 weeks		Oct. 2, 1922 after $12\frac{1}{2}$ weeks		Jan. 4, 1923 after $25\frac{1}{2}$ weeks	
	5 w. $35^{\circ}$	5. w. $17^{\circ}$	5 w. $35^{\circ} + 7\frac{1}{2}$ w. $17^{\circ}$	$12\frac{1}{2}$ w. $17^{\circ}$	5 w. $35^{\circ} + 7\frac{1}{2}$ w. $17^{\circ}$	$12\frac{1}{2}$ w. $17^{\circ}$
2.99	3.43	4.00		12.39	70.2	38.8

From this table 2 we see, that in the bulbs exposed to 5 weeks  $17^{\circ}$  the length was more after 5 weeks, that after an exposure to 5 weeks  $35^{\circ}$ , whilst on the other hand on January 4 it appears that the foliage-leaves in this latter exposure have grown twice as long as in those which had been constantly exposed to  $17^{\circ}$ .

§ 3. *On the number of cells in the longitudinal axis of the outer foliage-leaves.*

Let us now consider the result of counting the cells in connection with table 3.

The number of cells found in each leaf is given in the third column. There the number of cells has been given, lying in the spaces counted between the micrometer-divisions.

The number of spaces (= sps.) multiplied by 1.73 gives the lengths of the foliage-leaves in mms. This also enables us to compare with one another the foliage-leaves cut in each treatment. It may be observed that the number of cells of these foliage-leaves shows very little mutual variation.

This must probably be attributed to the accurate selection of the specimens to be treated before starting the experiments, besides to the fact that afterwards only those leaves were cut the length of which corresponded as much as possible with the macroscopically determined average. On account of the very slight variation in these figures and the very conspicuous results, the cutting of only three foliage-leaves on July 4, August 9 and September 30 proved sufficient.

On discussing the results we shall always refer to the *averages* of these



TABLE 3. Number of cells counted (Sps. = spaces in the micrometer.)

Date	Exposure	Number of cells counted in each foliage-leaf	Macroscopic length of each foliage-leaf in mms (See table 1)	Average number of cells
Juli 4	Beginning of the experiments	18 sps. = 3.11 mms. = 71 cells	3.2	<b>3.16 mms. = 69.3 cells</b>
		18 sps. = 3.11 mms. = 70 cells	3.3	
		17 sps. = 2.94 mms. = 67 cells	3.0	
Aug. 9	5 w. 35°	19 sps. = 3.29 mms. = 71 cells	3.2	<b>3.3 mms. = 73.5 cells</b>
		19 sps. = 3.29 mms. = 75 cells	3.2	
		19.6 sps. = 3.63 mms. = 75 cells	3.5	
Aug. 9	5 w. 17°	21.5 sps. = 3.72 mms. = 87 cells	3.5	<b>3.65 mms. = 92.5 cells</b>
		24.0 sps. = 4.15 mms. = 105 cells	3.7	
		21.0 sps. = 3.63 mms. = 86 cells	3.5	
Sept. 30	5 w. 35° + 7½ w. 17°	127.5 sps. = 22.05 mms. = 418 cells	23.0	<b>23 mms. = 426 cells</b>
		120.6 sps. = 20.86 mms. = 437 cells	23.2	
		123.7 sps. = 21.40 mms. = 423 cells	22.9	
Sept. 30	12½ w. 17°	62 sps. = 10.72 mms. = 225 cells	11.1	<b>11.06 mms. = 233 cells</b>
		67 sps. = 11.59 mms. = 240 cells	10.9	
		62.5 sps. = 10.81 mms. = 233 cells	11.2	
Febr. 11	5 w. 35° + 7½ w. 17°	1039.6 sps. = 179.8 mms. = 1506.5 c.	178.0	<b>178 mms. = 1506.5 cells</b>
		460 sps. = 79.58 mms. = 1036 cells	83.0	
		458 sps. = 79.28 mms. = 968.5 cells	82.9	

figures; we put them in the last column. In this column we also find the average length in mms. In every case it was computed from the macroscopic length of the foliage-leaves from each treatment, which were destined for counting the number of cells. These figures give a more correct picture of the length than the figures of the converted number of spaces in the micrometer. In consequence of the continual tapping of the sections and the tight squeezing between elder-piths the cells did not quite recover their initial shape: now the length got slightly shorter, now slightly longer. But of course this did not influence the *number* of cells in the whole length. The number of mms. and spaces in column 3 was only given in order to

give the reader a survey of the figures obtained. To judge of the exact number of cells in the lengths the macroscopically measured lengths of the loose foliage-leaves (column 5) destined for counting the cells were always taken as average lengths.

On starting the experiments (July 4) we counted 69.3 cells in 3.16 mms.

As early as August 9 some difference in the two treatments is to be noticed. The leaves exposed to  $17^{\circ}$  for 5 weeks, contained more cells than those exposed to  $35^{\circ}$  for 5 weeks. In this latter treatment there probably have been added some cells since July 4, but the exposure to  $17^{\circ}$  shows a greater increase. What was suggested as probable in the macroscopic measuring (the increase in length p. 199) is beyond dispute on *counting* the cells. In these weeks the number increased from 69.3 to 92.5 cells.

On Sept. 30 i.e. after  $12\frac{1}{2}$  weeks we get quite a different picture. With the macroscopic measurements we already found that on this date the preliminary treatment 5 weeks  $35^{\circ}$  gives to the foliage-leaves twice the length of those exposed to 5 weeks  $17^{\circ}$ , but *now it appears that the number of cells is also nearly twice as great*. The greater length, therefore, is not due to longer cells, but to a greater number of cells: *so the high temperature proves to have a very favorable effect on the cell-division*; not however while the bulbs are in this high temperature, but especially after that period, i.e. as a very favorable after-effect. For while the bulbs were kept in  $35^{\circ}$ , the number of cells in the longitudinal direction was smaller than in the bulbs exposed to  $17^{\circ}$ ; when however the two exposed groups have been in  $17^{\circ}$  for a long time, we begin to notice the favorable aftereffect of  $35^{\circ}$  on the cell-division in the longitudinal direction.

On the *thickness* of the foliage-leaves these various temperatures have no influence, for it appeared that the thickness of the foliage-leaves always consisted of the same number of cells, viz. varied between 16—18.

The cutting of the leaves on Sept. 30 already answered the question, what the greater length of the foliage-leaves exposed to  $35^{\circ}$  for 5 weeks, was due to. For the sake of completeness some leaves were cut on February 11 after all. The leaves of the group exposed to  $35^{\circ}$ — $17^{\circ}$  already having a length of 18 cms. on February 11, cutting 1 foliage-leaf of this lot was considered sufficient. The cutting of 2—3 foliage-leaves and the counting in parts would have taken up an exceedingly long time, whilst the favorable effect of  $35^{\circ}$  on the cell-formation was an ascertained fact. Of the group  $12\frac{1}{2}$  weeks  $17^{\circ}$  the cells of two foliage-leaves were counted on this date.

From table 3 last column we see, that on February 11 the group  $35^{\circ}$ — $17^{\circ}$  still has  $1\frac{1}{2} \times$  as many cells in the longitudinal direction of its foliage-leaves, as the group "always  $17^{\circ}$ ". The ratio of the number of cells of the groups is somewhat altered with respect to the previous observation. This is owing to the fact the group  $12\frac{1}{2}$  weeks  $17^{\circ}$  is no more lagging behind so far in the formation of the cells. This we may read from table 4.

TABLE 4. The average number of cells formed per week.

Group	July 4 — Aug. 9	Aug. 9—Sept. 30	Sept. 30 — Febr. 11
5 w. 35° + 7½ w. 17°	0.8	47	60
12½ weeks 17°	4.6	18.7	42.1

It has been computed how many cells have been formed *per week* in the two groups in the periods July 4—August 9, August 9—September 30 and September 30—February 11. This was done by dividing each number added in those periods (Table 3) by the number of weeks. These figures give us a standard, on account of their being formed by the dividing zone at the base of the leaf and are therefore not influenced by the length or the number of cells of the foliage-leaf. It is clear that the increase in cell-formation does not occur by leaps and bounds, but gradually.

Now we see from table 4, that in the group 35°—17° from August 9—September 30, 47 cells are formed per week, whilst from September 30—February 11, 60 cells are added per week. The lot constantly exposed to 17° first forms but 18.7 cells in those periods, afterwards 42 cells per week. In both groups the cell-formation is still increasing, but in the weeks from September 30—February 11 the group "always 17°" shows a greater increase compared with the previous weeks. The number of cells added per week in 35°—17° and the number of "always 17°" from August 9—September 30 are in the ratio of  $\pm 5 : 2$ , while in the next period that ratio has become  $10 : 7$ . Yet it appears, that the high temperature 35° still has a *favorable after-effect*, since this group adds 60 cells per week, and the group "always 17°" does not add more than 42 cells per week. The fact that the lot "always 17°" catches up a little in the later months may be due to the shifting of the optimum for the length of the foliage-leaves from higher to lower temperatures (see BLAAUW 1924, p. 16) in the later weeks.

On February 11 it moreover appears, that the group 35°—17° has slightly larger cells in the upper part of the leaf, from which it may be concluded, that the cells are stretched slightly more than in "always 17°". In order to get a survey of the size of the cells, I subjoin a successive series of a foliage-leaf of 35°—17° and one of "always 17°".

By comparing these figures we may prove : 1°. that preliminary treatment with 35° 5 wks makes the organ twice as long as only in 17°; 2°. that while on Sept. 30 there are indeed  $2 \times$  as many cells, months later (Febr. 11) the length *does* remain twice as much after stretching, but the number of cells is but  $1\frac{1}{2} \times$  as much; 3°. that this follows from the fact that the cells higher up in the leaf have become larger after 5 wks 35° + 7½ wks 17° than after "12½ wks 17°" (so that there are fewer in an equal distance).

It would be interesting to trace when the high temperature which now has a favorable effect in these experiments, will become injurious to the

TABLE 5.

Febr. 11 5 wk. 35° + 7½ wk. 17°				Febr. 11 12½ wk. 17°			
base	29.3 sps. *)	=	148.5 cells	base	38.6 sps.	=	171 cells
	49.5 "	=	209. — "		59. — "	=	224 "
	57.8 "	=	170. — "		52.4 "	=	145 "
	64. — "	=	124. — "		53.3 "	=	106.5 "
	58. — "	=	100. — "		66.5 "	=	105.5 "
	66. — "	=	87. — "		66. — "	=	102. — "
	67.5 "	=	84. — "		65.2 "	=	92.5 "
	67. — "	=	62.5 "	apex	59. — "	=	89.5 "
	66.5 "	=	55. — "	460.0 sps. = 1036. — cells			
	66.5 "	=	56.5 "				
	67. — "	=	58. — "				
	62. — "	=	54. — "				
	67. — "	=	62. — "				
	69. — "	=	66. — "				
	64.5 "	=	59. — "				
	57. — "	=	52. — "				
apex	61. — "	=	59. — "				
1039.6 sps. = 1506.5 cells							

\*) sps. = spaces of the eye-piece micrometer

cell-formation and have an inhibitory effect. For this subject we once more refer to BLAAUW (1924), who in *longer exposure* to 35° observed a *decrease* in length in the foliage-leaves. In a group, exposed to 5 wks 35° + 7½ wks 17° the length in Jan. was 70.2 mms.; in a lot exposed to 8 wks 35° + 4½ 17°, the length in Jan. was 63.4 mms., whereas the group exposed to 12½ wks 35° did not attain a greater length than 34.2 mms. in January.

Our research having proved, that the high temperature gives the difference in length through the different number of cells formed afterwards, it might be worth while transferring lots from a group of bulbs, exposed already to 35° for 5 weeks, at regular rather short intervals to 17°, and next fix all of them on a certain date. Then we could determine by counting the cells, when inhibition begins.

Wageningen, Nov. 1926.

Laboratory for  
Plant-physiological Research.



## LITERATURE.

BLAAUW, A. H. (1924). The results of the temperature during flower-formation for the whole Hyacinth: First part. Verhandel. Koninkl. Akademie v. Wetensch. Amsterdam 2de Sectie, Dl. 23, N<sup>o</sup>. 4 en Meded. N<sup>o</sup>. 10 v. h. Labor. v. Plantenphysiol. Onderzoek, Wageningen.

VERSLUYS, MARTHA C. (1925). The mode of growing of foliage-leaves, sheath-leaves and bulb-disc in *Hyacinthus orientalis*. Recueil des travaux botaniques néerlandais Vol. 22, en Meded. N<sup>o</sup>. 14 v. h. Laborat. v. Plantenphysiol. Onderzoek, Wageningen.

---

**Botany.** — *On the atmospheric humidity during the flower-formation of the Hyacinth.* By Prof. A. H. BLAAUW. (Communication N<sup>o</sup>. 23. Laboratory for Plant-physiological Research, Wageningen.)

(Communicated at the meeting of January 29, 1927).

### § 1. *Introduction.*

Between the lifting and planting of the Hyacinths, i.e. from the beginning of July to the beginning of October, lies the important period, in which the bulbs pass on to flower-formation and in which a good flowering for the next year, — or if desired the early flowering in December — entirely depends on the adequate treatment. With regard to this we have first of all to deal with the temperature as most important factor. Hence the influence of the temperature was first investigated into, and part of these researches were already published, whilst further papers on this subject will follow after being worked out.

The only other important factor might be the hygrometric condition of the air. The bulbs have just received the assimilation-products from the leaves. It is often said — and frequently not unjustly — that the concentration of the assimilation-products enables a plant, during vigorous cell-division, to pass on to flower-formation. This makes us ask ourselves whether, when the concentration is forwarded by a dry atmosphere, so that the bulbs evaporate a great deal of water, the assimilation-products do not accumulate more inward and in this way the atmospheric humidity may influence the flower-formation.

In practice the growers often talk of "dry-storing" and the question rises whether drought is really favorable for the growing plant, in what degree and whether from July to October the drought should be equal or varied. Questions, therefore, essential both to the theory of flower-formation and leaf-development and to the application.

Based on experiments fully described and on the fact that the temperature is the most important factor after all, we stick to the optimal temperature-treatment for field-cultures; after the lifting till ca Sept. 1 (ca 8 weeks) 25° to 26°, next till the planting (ca 4½ weeks) 17° at most.

After having repeated experiments on the hygrometric condition of the air for 3 summers, I shall communicate their results here.

For reading the hygrometric condition of the air, expressed in percents of the saturation, we chose of various makes or systems as the most practical for our use the hair-hygrometers of RICHARD in the shape of an alarmclock (Hygromètre à cadran; section of dial 10 cms.). There are those among them which are unsuitable for some reason; therefore a

choice should be made and those taken into use which are sensitive enough and which in spite of a change of temperature or humidity, when reduced to the same circumstances almost point to the same hygrometric condition. The correct height of the Hygrometer is regulated by repeated comparison with the Assmanns-Aspiration-psychrometer, i.e. "gauged", which regulation is easily performed by adjusting the hand.

Next 4 experiments were compared each year, viz. the first hot period (26°) moist, the second cooler period (17°) dry, indicated *VD*, or first dry then moist (*DV*), or both periods dry (*DD*) or both periods moist (*VV*).

The bulbs were put in glass or china dishes in two zinc boxes. Those boxes were put on their sides, the open "upper side" turned towards the front, slightly sloping backwards, a glass-plate against them. The dry box contained (besides the bulbs and a hygrometer) a dish of unslacked lime for drying, which was renewed as soon as the humidity approached the fixed upper limit. The moist box was kept at the desired moisture by a little water at the bottom or by an evaporation-tube (of a radiator of the central heating). The chinks between the glass and zinc box were covered with moist cloths if necessary.

For the drought (*D*) the hygrometric condition was kept at 30—40 % in all three summers, for the moisture (*V*) in 1923 first at 80—90 %, in 1924 and 1925 even at 90—100 %, as a rule 95—100 %. On purpose strong contrasts were chosen. But as moderately moist and moderately dry might be more favorable than one of these four extremes, after the result of 1923 a 5<sup>th</sup> group was added in 1924 and 1925, in which fairly moist amounted to 70—80 % and fairly dry to 50—60 %. This 5<sup>th</sup> group has been indicated as "control group" in the following tables.

In the very moist treatment there naturally occurred mould, only however on the totally exhausted outer scales and the bottom disc-layer. On account of this these bulbs were cleaned a few times with a cloth. Rotting or injury to living parts did not occur in spite of the very high moisture-content.

## § 2. *Experiments in the year 1923—1924.*

In the summer of 1923 2 year old bulbs were chosen of a circumference of 80—90 cms. By that time the different experimental groups were selected only according to the circumference and not yet — as always happened in later years — divided into groups of equal weight.

Consequently in October after the treatment there was a certain variation between the weights of the 30 bulbs (per experiment), which variation is arbitrary and cannot just be attributed to the previous treatment (See Tab. 1).

Of the 30 bulbs treated per experiment some 9 were lifted afterwards in order to trace whether the humidity had any effect on the root-system.

Various data on the root-system, also in connection with different circumstances, have been examined and will be published in a separate treatise by Mrs. M. C. TROOST—VERSLUYS.

### Results 1923—1924.

Though in February 1924 the group *VD* came up somewhat later than the three other groups, yet on March 26 all four groups were perfectly equal. Likewise on April 22<sup>nd</sup> all groups were uniform and in full flower, so that not the slightest distinction could be made as a result of the treatments.

An influence on the number of foliage-leaves may hardly be expected, since on lifting the foliage-leaf-formation ceases and at 26° the growing-point passes on to flower-formation. Yet with so severe a drought (e.g. in *DD* and *DV*) contrasted with a high moisture-content (in *VD* and *VV*) the leaf-formation might cease at different points of time, and at any rate an influence on the shooting or non-shooting of the last-formed leaflets was not unthinkable. In Tab. 1, first column, we see that after the treatments of 1923 the number of shooted foliage-leaves computed for 10 bulbs was identical (38) averagely in three groups, in *VV* slightly smaller (36), but this difference of upwards of 5 % may be quite accidental. In *VV*, therefore, the influence on this is at most very slight; in the three other cases the shooting is at all events the same. We shall revert to this in the further experiments.

TABLE 1. Treated summer 1923, beginning of July 80—90 mms.

Treatment	Weight per 20 bulbs. Beginning of Oct. 1923 after treatment	Number of shooted foliage-leaves per 10 bulbs	Number of flowers per 10 plants ( $n = 30$ )	Average length of the foliage above ground in mms.	Weight per 20 bulbs July 6, 1924	Average circumference per bulb July 6, 1924.
1	2	3	4	5	6	7
VV	199 Grams	36	62.4 ( $\pm 2.7$ )	334.6 ( $\pm 4.3$ )	640	125.8 mms. ( $\pm 2.0$ )
DD	201 "	38	62.0 ( $\pm 2.4$ )	313.9 ( $\pm 7.4$ )	586	118.1 " ( $\pm 2.2$ )
DV	209 "	38	63.7 ( $\pm 2.5$ )	311.9 ( $\pm 6.5$ )	596	121.9 " ( $\pm 2.1$ )
VD	194 "	38	66.0 ( $\pm 2.3$ )	342.4 ( $\pm 3.5$ )	702	131.7 " ( $\pm 1.6$ )

In this first year there initially seemed to be not the slightest difference between the four groups. The four lots flowered particularly uniformly. The average number of flowers per cluster is still slight in these small bulbs. Column 4 gives the number per 10 plants as an average from 30 specimens after the different moisture-treatment in the previous summer. That number is also equal in the four groups. True, after *VD* the number seems slightly higher, but the difference does not exceed the amount possible according to the mean errors.



At the beginning of May after the roping of the flowers the foliage was also perfectly equal. Not before the end of May and the beginning of June the length of the foliage-leaves in the various groups became unequal. At the end of May the length of the foliage was measured above ground. Column 5 shows that there certainly is a difference in length here, even though the mean error is taken into account. The two groups *VV* and especially *VD* as averages of 20 observations are clearly 2 and 3 cms. longer than those which were first treated dry.

While the leaves were still growing out in June, no further measures could be taken, because by that time the leaf-apexes begin to dry up, though the rest of the leaf is still assimilating. It is however more important to trace the effect of the assimilation-period in the increase of weight (or increase of circumference).

In Tab. 1 columns 6 and 7 show us the following :

The weight per 20 bulbs rather varies at the beginning of Oct. 1923 ; this, however, does not prove anything with regard to more or less evaporation, since in July the groups had not yet been divided according to equal weights as later on (see 1924 and 1925).

The difference in increase of the bulbs after different moisture-treatment (previously between lifting and planting) was so striking in 1923—1924, that we understood that these experiments ought to be continued and repeated. The very bulbs which chanced to have the lowest weight at the beginning of Oct., i.e. after the treatment and before the planting, have by far the greater weight on July 6, 1924. Those are the two groups which were kept extremely moist for two months after the lifting, whilst the greatest increase in weight was shown by the one that was kept very dry at 17° after that.

So it appears from four groups, *that at all events during the flower-forming period a very moist atmosphere has a more favorable effect on the increase in thickness of the bulbs than a dry atmosphere in July and August*, whilst no detriment e.g. concerning diseases was noticed. This result we ourselves had not surmised at all ; it already proves that the idea "dry-storing" often used in practice, is put in a peculiar light. We shall however first have to consider the results of following years. In Tab. I the average circumference on July 6 has also been given. From this the same results can be read as from the column of the weights of the four groups. Apparently the circumferences do not diverge so much as the weights or the increases of weight. But these two criteria form an essential point for comparison of magnitudes as we repeatedly want them. *With the weights we have to deal with a comparison of the heaviness of certain bodies, i.e. we compare with each other three-dimensional magnitudes. With the circumferences we only compare a certain linear measure of those same bodies.* Therefore on our comparing the circumferences the difference of the real contents will be much less conspicuous than on our comparing groups of bulbs by their weights. Hence after having exclusively worked

with circumferences in the first years, we now in particular compare the weights in our experiments, wherever this is possible.

Meanwhile we have already got a great number of data on weight and circumference of bulbs, so that afterwards we shall probably revert to the connection between these two magnitudes.

### § 3. *Experiments of the years 1924—1925 and 1925—1926.*

In this § 3 the results of 1924—1925 and 1925—1926 will be discussed together. The experiments were started on July 9, 1924 with bulbs having a circumference of 110—120 mms., weighing 518 grams per 20; in 1925 the experiments were started on July 3 with bulbs of 115—120 mms., i.e. selected within slightly narrower limits, weighing 531 grams per 20, so somewhat heavier than in 1924, which corresponds with the limits of circumference which were chosen 115—120 mms. instead of 110—120 mms.

As was already stated in the conclusion of § 1, a control-group has been added of fairly moist ( $25\frac{1}{2}^{\circ}$  C.) + fairly dry ( $17^{\circ}$  C.).

Mid-September 1924 in V V the roots had already shootd so far, that this group had to be planted in the field with great care.

Thus V V of 1924 was planted with shooting roots in the field upwards of 14 days earlier than all other groups. In 1925 this did not occur in V V, though the root-whorl was already much developed on Oct. 1.

In the spring of both years all five groups first mutually behaved similarly, just as the four groups in the first year did.

Here the question first rises whether there is any influence on the number of flowers. In April 1925 the average is 13 to 14 per cluster in the control-group, V V and V D, compared to 15 to 16 in D D and D V; taking the mean error into consideration we might decide here on a slight difference in favour of D D and D V.

TABLE 2. Number of flowers per cluster in April after the different moisture-treatments in the previous summer.

	Circumference 110—120 mms. April 1925	Circumference 115—120 mms. April 1926
Control-group	12.9 ( $\pm 0.5$ )	12.8 ( $\pm 0.4$ )
V V	13.3 ( $\pm 0.8$ )	12.0 ( $\pm 0.4$ )
D D	15.2 ( $\pm 0.6$ )	12.5 ( $\pm 0.5$ )
D V	15.8 ( $\pm 0.6$ )	12.1 ( $\pm 0.5$ )
V D	14.2 ( $\pm 0.7$ )	12.8 ( $\pm 0.4$ )

On our considering the experiment of 1925—1926, we see that the number of flowers of an average of 12 to 13 per cluster differs very little in all 5 groups. The mean errors should be taken into account — and it should likewise be borne in mind, that in the more accurately selected material the mean error (computed for groups of 20) is also visibly slighter.

After having repeated these experiments for 3 years, we must conclude, *that the atmospheric moisture (between 30 % and 100 %) has no noticeable effect on the number of flowers of the cluster in the flower-forming period.*

Let us now consider as with the experiments of 1923—1924 how many foliage-leaves shoot and assimilate in the spring. For 1925 and 1926 this is found in Table 3. In the 5 groups of 1924—1925 this number varies from 48 to 51, in 1925—1926 from 51—55 per 10 plants. This difference per 10 plants is certainly slight. In this respect too we must conclude

TABLE 3. Number of foliage-leaves shooted per 10 bulbs.

	1925	1926
Control-group	51	55
VV	48	51
DD	49	52
DV	50	52
VD	49	53

after comparison of the results of 3 years from 4 and 5 groups, *that the hygrometric condition of the air between 30 % and 100 % has no noticeable effect on the shooting of the number of young leaves already formed, which are to assimilate next spring.*

Moreover we see that *during the flowering and a short time after in April the foliage is uniform in the various groups.*

It was already mentioned in the first series of experiments, how in June 1924 at last a striking dissimilarity was observed, because finally the foliage of group *VD* grew a good deal longer, which was corroborated in the results, because the increase of weight in this group was by far the greatest.

Table 4 gives the average length of the foliage in the beginning of June 1925 and 1926. Already in this case with the leaf-lengths, but still more so from the subsequent tables 5 and 6 on the increase in weight it appears, that 1925 was a more favorable year for assimilation than 1926.

TABLE 4. Average length of the foliage in mms. above ground, beginning of June, after different moisture-treatments in the previous summer.

	Circumference 110—120 mms. June 1925	Circumference 115—120 mms. June 1926
Control-group	293.1 ( $\pm$ 4.5)	283.0 ( $\pm$ 5.1)
VV	315.8 ( $\pm$ 7.9)	260.5 ( $\pm$ 7.0)
DD	307.0 ( $\pm$ 7.4)	286.1 ( $\pm$ 5.8)
DV	288.0 ( $\pm$ 9.3)	275.5 ( $\pm$ 9.5)
VD	320.7 ( $\pm$ 5.2)	291.6 ( $\pm$ 5.8)

With regard to the Hyacinth it may make a great difference whether the winter lasts long and is followed by a warm, dry early summer, or that we have a rather early mild spring passing into a moderately warm and rather moist early summer. The assimilation-period, as it is already very limited for the Hyacinth may be very short or rather long in consequence of this.

Hence the result of experiments can be so divergent one year and an other, though we compare the effect of treatments applied to the bulbs in summer under completely controlled conditions.

In Tab. 4 we give the average length of the foliage in mms. above ground ca June 1. Little may be concluded from this. Considering the mean errors, the differences are rather slight in most cases. Yet in both

TABLE 5. Increase of weight in grams per 20 bulbs.  
Treated summer 1924. Circumference 1924: 110—120 mms.

	Weight 20 bulbs July 9, 1924	Weight 20 bulbs Sept. 30, 1924	Weight 20 bulbs July 8, 1925	Increase in a year
Contr.	518	448	933	415
VV	518	planted earlier see text	908	390
DD	518	421	890	372
DV	518	441	869	351
VD	518	433	941	423



TABLE 6. Increase of weight in grams per 20 bulbs.  
Treated summer 1925. Circumference 1925: 115—120 mms.

	Weight 20 bulbs July 3, 1925	Weight Sept. 30, 1925	Weight Juni 28, 1926	Increase in a year
Contr.	531	413	779	248
V V	531	449	718	187
D D	531	397	746	215
D V	531	422	788	257
V D	531	420	821	290

years the foliage is longest in *V D*. In 1926 *V V* yields a fairly low figure, while this group succeeded *V D* in foliage-length in 1924 and 1925. Taking all together we should not attach too much value to this measure of the leaf-lengths; we can only say that the phenomenon of 1924 "that after the treatment *V D* the leaves finally attain a somewhat greater length" is confirmed in 1925 and 1926, though it is not so striking as in 1924.

Let us finally consider the increase of weight in 1925 and 1926. First of all these tables 5 and 6 prove that with slight differences in leaf-lengths as in Tab. 4 we should be careful and that these are not quite parallel with the increase in thickness of the bulbs. Only in case of somewhat greater differences we may rely to some extent on the foliage-lengths. So we find that in 1925 and 1926 the groups with the longest and with the shortest foliage are likewise the best and the worst group as regards the weight.

In Table 6 attention should be paid first of all to the loss of weight of the bulbs directly after the treatment. As regards the evaporation, of course this depends greatly on the moisture or drought of the storage-atmosphere. Starting from 531 grams the group *V V* has lost 82 grams or 15 %, the group *D D* 25 % (in 1925 but 19 %), while the other groups lost an amount between those two extremes. We see that even with 90—100 % moisture the loss of weight though much slighter, is yet considerable (15 %). This loss of weight will for the greater part be due to evaporation, but will partly be owing to oxydation. That part of the loss of weight which is due to respiration, may be expected to remain constant in this different atmospheric moisture but uniform temperature-treatment.

Both in 1925 and 1926 this increase of weight at the end of the assimilation-period is greatest after the treatment *V D*, though in 1924 the differences were greater (see Tab. 1). Of the remaining groups little can be said with certainty about the three years. In 1925 the control-group fairly

moist-fairly dry produces a favorable effect, in 1926 a rather favorable, it does not belong to the 2 worst groups. In the three years the worst figure is successively yielded by *DD*, *DV* and *VV*; the least increase but one by the groups *DV*, *DD* and *DD*. While in 1926 *VV* increases strikingly little, this group was fairly favorable in 1924 and 1925. So we see that the groups *DD* and *DV* yield one of the two lowest figures 5 of the 6 times in those three years, the group *VV* but once, while *VD* gives the greatest increase all the 3 times.

#### § 4. Conclusion.

We have exposed Hyacinth-bulbs in one and the same temperature-treatment to very moist and very dry conditions and repeated this three years running.

The hygrometric condition (30 %—100 %) of the surrounding air in the period of storing has *no influence on the number of leaves that can shoot the next year.*

The hygrometric condition of the air during that flower-forming period has *no influence on the number and the good development of the flowers.*

Finally the applied moisture is noticeable next summer in the final length of the foliage, as very moist 25° C. followed by very dry 17° C. grows out a little more than all other groups, which was corroborated all the three years.

Probably it may be considered as an additional result that all the three years after this treatment the greatest increase of weight was found, exceeding the results of the other treatments sometimes more, sometimes less. Besides the application of very dry in the first months is by no means favorable, so that dry-dry and dry-moist are far behind moist-dry in increase of weight. Moist-moist is now fairly good, now unfavorable and should be avoided, because of the danger that the roots will shoot untimely in 17° in September.

A moisture of 90—100 % followed by 20—40 % is difficult to apply on a large scale. In great quantities the growing mouldy and the decay of old bulb-rests might yield a too great danger for the living parts of the bulb (which however could not be observed by us in small quantities). Moreover a drought of 30—40 % moisture is difficult to maintain in the large stores.

After the lifting of the bulbs we keep our rooms dry for a couple of days, until sand and root-rests get loose from the bulbs. next we raise the temperature to 26° C. and the moisture to 70—80 % for 8 weeks for the field-cultures. After 8 weeks we transfer to 17° C. with a moisture of 50—55 %.

Though on applying fairly moist and fairly dry the advantage of very moist—very dry is partly given up (see control-group in tables 5 and 6), the difficulties of applying very moist and very dry on a large scale as in our experiments, are avoided in that way.

Thus the hygrometric condition cannot act a great part in the application to considerable quantities. A moisture of 90—100 % followed by 30 % would certainly produce a greater increase of weight, but because of the difficulties of application we shall have to be satisfied with fairly moist (70—80 %) followed by fairly dry (50—55 %).

January 1927.

---

**Mathematics.** — *On metric connexions with absolute parallelism.* By E. BORTOLOTTI. (Communicated by Prof. JAN DE VRIES.)

(Communicated at the meeting of February 26, 1927).

I wish to indicate in this paper some results <sup>1)</sup> concerning the metric connexions <sup>2)</sup> with zero curvature in an  $n$ -dimensional RIEMANNIAN manifold ( $V_n$ ). Like results are in connexion with those which CARTAN and SCHOUTEN have established in two papers, published here in 1926. <sup>3)</sup>

CARTAN and SCHOUTEN have determined the RIEMANNIAN spaces admitting some metric connexions with absolute parallelism (namely with zero curvature), which have the same geodesics as the RIEMANNIAN connexion (by CARTAN: connexion euclidienne de M. LEVI-CIVITA).

Omitting the last condition, in an arbitrary  $V_n$  we establish an absolute parallelism, with respect to an arbitrary orthogonal ennuple  $\dot{X}^\nu$  <sup>4)</sup> in  $V_n$ , if we assume that *all vectors parallel to one another make the same angles with the curves of the congruences  $\dot{X}^\nu$* . This parallelism has been studied, from an analytical point of view, by G. VITALI <sup>5)</sup>; but previously also R. WEITZENBÖCK <sup>6)</sup> has considered the tensorial differentiation to which that parallelism corresponds.

<sup>1)</sup> Which have been more developed in an article ("*Parallelismi assoluti nelle  $V_n$  Riemanniane*") of the "Atti Ist. Veneto", t. 86. p. II, 1926—1927, p. 455—465.

<sup>2)</sup> I call here a "*metric connexion*" the connexion which arises from an "*Ueberschiebungsinvariante metrische Uebertragung*" (J. A. SCHOUTEN, *Der Ricci-Kalkül*, Berlin, Springer 1924, p. 75). E. CARTAN calls this connexion a "*connexion euclidienne*"; while CARTAN's "*connexion métrique*" (which generalizes the WEYL's "*metrische Zusammenhang*"), corresponds to SCHOUTEN's "*Ueberschiebungsinvariante konforme Uebertragung*".

<sup>3)</sup> E. CARTAN and J. A. SCHOUTEN: "*On the Geometry of the Group-manifold of simple and semi-simple groups*", Proceedings 29, p. 803—815, and "*On RIEMANNIAN Geometries admitting an absolute parallelism*". *ibid.* p. 933—946. S. also E. CARTAN: "*La Géométrie des groupes de transformations*". (Journal de Mathématique, t. VI, 1927, p. 1—119) part. p. 49—54.

<sup>4)</sup>  $\dot{X}^\nu$  ( $i = 1, 2, \dots, n$ ) denote here the contravariant components of  $n$  mutually orthogonal (unit-) vectors-field in  $V_n$ .

<sup>5)</sup> G. VITALI: "*Una derivazione covariante formata coll'ausilio di  $n$  sistemi covarianti del 1° ordine*". (Atti Soc. Ligustica, vol. II, 1924, pag. 248—253).

<sup>6)</sup> R. WEITZENBÖCK: "*Invariantentheorie*". P. Noordhoff, Groningen, 1923, p. 320. S. likewise SCHOUTEN, *Ricci-Kalkül*, p. 101 (Aufgabe 1); G. J. C. GRISS, "*Differential-invarianten von Systemen von Vektoren*". Acad. Proefschrift; P. NOORDHOFF, Groningen. 1925), p. 10—14.



Let us call a WEITZENBÖCK-VITALI's connexion the metric one which arises from that parallelism. I have remarked that the equations of such a connexion may be expressed in the form:

$$\nabla_{\mu}^0 \xi^{\nu} = \nabla_{\mu}^0 \xi^{\nu} + T_{\lambda\mu}^{\cdot\cdot\nu} \xi^{\lambda} \quad (1)$$

where  $\nabla$  is the symbol of RICCI's covariant derivative, and  $T_{\lambda\mu}^{\cdot\cdot\nu}$  is given by

$$T_{\lambda\mu}^{\cdot\cdot\nu} = \sum_{i,j,l}^n \gamma_{ijl} X^i X^j X_{\mu}^l, \quad (2)$$

where  $\gamma_{ijl}$  are the coefficients of rotation of the ennuple  $X^{\lambda}$  <sup>8)</sup>.

Let us now assume that, in form. (1),  $T_{\lambda\mu\nu}$  denotes an arbitrary affnor, skew-symmetric with respect to  $\lambda, \nu$ . Then by (1) is represented the most general metric connexion in  $V_n$ .

The necessary and sufficient condition that there exists an ennuple  $X^{\lambda}$ , with respect to which  $T_{\lambda\mu}^{\cdot\cdot\nu}$  may be expressed in form (2), is that

$$R_{\omega\lambda\mu}^{\cdot\cdot\cdot\nu} = R_{\omega\lambda\mu}^{\cdot\cdot\cdot\nu} - \left\{ \nabla_{\omega}^0 T_{\lambda\mu}^{\cdot\cdot\nu} - \nabla_{\mu}^0 T_{\lambda\omega}^{\cdot\cdot\nu} + T_{\lambda\mu}^{\cdot\cdot\tau} T_{\tau\omega}^{\cdot\cdot\nu} - T_{\lambda\omega}^{\cdot\cdot\tau} T_{\tau\mu}^{\cdot\cdot\nu} \right\} = 0, \quad (3)$$

where  $R_{\omega\lambda\mu}^{\cdot\cdot\cdot\nu}$ ,  $R_{\omega\lambda\mu}^{\cdot\cdot\cdot\nu}$  are the RIEMANN-CHRISTOFFEL's affnors of the connexion (1), and of the RIEMANNIAN one in  $V_n$ .

Consequently:

*The most general metric connexion with zero curvature in a RIEMANNIAN  $V_n$  is the WEITZENBÖCK-VITALI's connexion with respect to an arbitrary orthogonal ennuple in  $V_n$ .*

Besides we have:

*The geodesics of an arbitrary metric connexion with zero curvature in  $V_n$  may be ranged in  $\infty^{\frac{n(n-1)}{2}}$  orthogonal ennuples, with respect to any of which the supposed connexion is a WEITZENBÖCK-VITALI's one.*

In particular, the connexions which have been studied in the above cited CARTAN and SCHOUTEN's papers, are WEITZENBÖCK-VITALI's connexions with respect to some orthogonal ennuples with skew-symmetric coefficients of rotation.

<sup>7)</sup> I make use of some notations of the "Ricci-Kalkül" of SCHOUTEN, S. pag. 63, 73, 83.  $\Delta_{\mu} \xi^{\nu}$  is called by WEITZENBÖCK the "kovariante Ableitung von  $\xi$  bezüglich der  $n$  Formen  $X^1, X^2, \dots, X^n$ "; by VITALI, the "primo sistema derivato covariante di  $\xi$ , rispetto agli  $n$  sistemi  $X^{\lambda}$ ".

<sup>8)</sup> RICCI et LEVI-CIVITA: "Méthodes de Calcul différentiel absolu et leur applications" (Mathematische Annalen, t. 54 (1900), p. 125—201) p. 149. It may be observed that from (2) arises a new signification for the coefficients of rotation of an orthogonal ennuple in  $V_n$ .

Let us say that an  $n$ -tuply system of mutually orthogonal congruences in  $V_n$  is a *canonical* one, where  $n-1$  (whatsoever) of these congruences are the (RICCI's) canonical congruences<sup>9)</sup> with respect to remainder one. Then we have:

*The RIEMANNIAN, CARTAN and SCHOUTEN's spaces (that is: the spaces admitting some metric connexions with zero curvature, and with the same geodesics as the RIEMANNIAN connexion) are characterized by possessing an  $n$ -tuply canonical system of orthogonal geodesic congruences; and, therefore, infinite ones, two of which (inversely like, corresponding to the two opposite signs of  $\gamma_{ij}$ , or  $T_{\lambda\mu\nu}$ ) are (generally) determined by choosing an orthogonal ennuple of directions in a point of the space.*

---

<sup>9)</sup> RICCI et LEVI-CIVITA: "*Méthodes, . . .*", pag. 154.

**Mathematics.** — *Ueber Syzygien bei sechs binären Linearformen.*  
By R. WEITZENBÖCK.

(Communicated at the meeting of January 29, 1927).

Wir zeigen in § 1 dass jede irreduzible  $p$ -Relation der  $G_d$ -Koordinaten  $p_{ikl\dots}$  in einem  $G_n$  ( $=$ lineares Gebiet  $n$ -ter Stufe oder linearer, projektiver,  $(n-1)$ -dimensionaler Raum) einem invarianten Gleichungssystem angehört, das durch Nullsetzen einer Komitante  $K$  des alternierenden Tensors  $p_{ikl\dots}$  entsteht. In § 2 wird dies für die quadratischen  $p$ -Relationen näher ausgeführt.

Die  $p$ -Relationen der  $G_d$ -Koordinaten in einem  $G_n$  sind Syzygien  $S=0$  zwischen den Invarianten  $J$  von  $n$  Punkten eines  $G_d$ . Umgekehrt ist auch jedes  $S=0$  eine  $p$ -Relation und die Struktur dieser Gleichungen wird aufgedeckt durch die Ermittlung aller *irreduziblen* Syzygien  $S$  und die dazu gehörige *Syzygienkette*.

Dies ist bisher bekannt für  $d=2$ ,  $n=5$  (§ 3). Wir behandeln hier den Fall  $d=2$ ,  $n=6$ , wo die Verhältnisse schon bedeutend komplizierter sind. Die zugehörige Syzygienkette bricht hier nach der vierten Art ab, d.h. es gibt noch (einundzwanzig) irreduzible Syzygien vierter Art aber keine von fünfter Art mehr (§§ 4 bis 9).

### § 1. Irreduzible $p$ -Relationen.

Es seien  $x, y, \dots, z$   $d$  linear-unabhängige Punkte eines  $G_d$  im  $G_n$  ( $G_m =$ linearen, projektiver,  $(m-1)$ -dimensionaler Raum), deren homogene Koordinaten die Elemente der Matrix

$$\mathfrak{M} = \begin{vmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ \dots & \dots & \dots & \dots \\ z_1 & z_2 & \dots & z_n \end{vmatrix} \dots \dots \dots (1)$$

bilden. Die  $d$ -reihigen Determinanten.

$$p_{i_1 i_2 \dots i_d} = (xy \dots z)_{i_1 i_2 \dots i_d} = (i_1 i_2 \dots i_d) \dots \dots \dots (2)$$

aus  $\mathfrak{M}$  sind die  $\binom{n}{d}$  homogenen „Punktkoordinaten des  $G_d$ “. Sie sind verknüpft durch eine Reihe von Gleichungen, die wir  $p$ -Relationen  $R=0$  nennen, wenn  $R$  ein Polynom der  $p_{ikl\dots}$  ist, das die beiden Bedingungen erfüllt: a)  $R \not\equiv 0$   $\{p_{ikl\dots}\}$ , d. h.  $R$  ist nicht Null, wenn die  $p_{ikl\dots}$  als unab-

hängige Veränderliche betrachtet werden:  $\beta) R \equiv 0 \{x, y, \dots, z\}$ , d.h.  $R$  verschwindet identisch, wenn die  $p_{ikl\dots}$  durch die  $x_i, y_k, \dots$  ausgedrückt werden.

Wir wollen weiters von einer *irreduziblen*  $p$ -Relation  $R=0$  sprechen, wenn erstens  $R$  nicht als Summe  $R_1 + R_2 + \dots$  geschrieben werden kann, wo schon jedes  $R_i=0$  eine  $p$ -Relation darstellt und wenn zweitens  $R$  nicht (im Körper der rationalen Zahlen) in Faktoren zerlegbar ist:  $R=R_1 R_2$ , wo schon  $R_1=0$  oder  $R_2=0$  eine  $p$ -Relation ist.

Die einfachsten irreduziblen  $p$ -Relationen sind die *quadratischen*  $p$ -Relationen  $\Pi=0$ . Hierbei ist der allgemeine Typus von  $\Pi$  gegeben durch:

$$\Pi = (i_1 i_2 \dots i_d)(k_1 k_2 \dots k_d) - (k_1 i_2 \dots i_d)(i_1 k_2 \dots k_d) + \dots + (-1)^d (k_1 i_1 i_2 \dots i_{d-1})(i_d k_2 \dots k_d) = 0 \quad (3)$$

und es besteht der Satz <sup>1)</sup>, dass jede  $p$ -Relation  $R=0$  sich auf die Gestalt

$$R = \sum A_\nu \Pi_\nu = A_1 \Pi_1 + A_2 \Pi_2 + \dots = 0 \quad (4)$$

bringen lässt, wo die  $A_\nu$  Polynome der  $p_{ikl\dots}$  sind.

Die Gleichungen  $\Pi=0$  sind notwendig und hinreichend dafür, dass  $\binom{n}{d}$  Grössen  $p_{ikl\dots}$  als Punktkoordinaten eines  $G_d$  im  $G_n$  betrachtet werden können.

Wir wollen jetzt zeigen, dass jede irreduzible  $p$ -Relation  $R=0$  aus einer Komitante  $K$  des schiefsymmetrischen Tensors  $p_{ikl\dots}$  abgeleitet werden kann durch Spezialisierung der in  $K$  auftretenden Reihen  $\xi, \eta, \dots, u', v', \dots$ .

Hiezu haben wir vor allem nachzuweisen, dass jedes irreduzible  $R$  homogen in den  $p_{ikl\dots}$  und ausserdem homogen in allen auftretenden Indizes  $i, k, l, \dots$  ist. Wäre nämlich  $R$  nicht homogen in den  $p_{ikl\dots}$ , dann würde  $R$  additiv zerfallen in homogene Bestandteile  $R=R_1 + R_2 + \dots$ . Ersetzen wir hier  $x_i$  durch  $\lambda x_i$ , dann erscheint jedes  $p_{ikl\dots}$  mit  $\lambda$  multipliziert und aus  $R_1 + R_2 + \dots = 0$  wird  $\lambda^{m_1} R_1 + \lambda^{m_2} R_2 + \dots = 0$  mit  $m_i \neq m_k$ . Also wäre  $R_i=0$  bereits eine  $p$ -Relation.

Die Indizeshomogenität bei einer irreduziblen  $p$ -Relation beweist man genau so, indem man  $x_i, y_i, \dots, z_i$  durch  $\lambda x_i, \lambda y_i, \dots, \lambda z_i$  ( $i$  fest) ersetzt.

Jetzt leiten wir aus  $R$  ein Polynom  $R'$  ab, das ebensoviele untereinander äquivalente Reihen  $p_{ikl\dots}, q_{ikl\dots}, r_{ikl\dots}, \dots$  linear enthält, als der Grad von  $R$  in den  $p_{ikl\dots}$  beträgt. Wir bilden hiezu die Polaren ( $m = \text{Grad von } R \text{ in den } p_{ikl\dots}$ ):

$$R_1 = \frac{1}{m} \sum \frac{\partial R}{\partial p_{ikl\dots}} q_{ikl\dots}, \quad R_2 = \frac{1}{m-1} \sum \frac{\partial R_1}{\partial p_{ikl\dots}} r_{ikl\dots}, \dots, R_{m-1} = R'.$$

Wir haben dann

$$(R')_{p_{ikl\dots} = q_{ikl\dots} = \dots} = R \quad (5)$$

<sup>1)</sup> Vgl. einen demnächst in den Mathem. Annalen (1927) erscheinenden Beweis.



Da  $R'$  linear in jeder Reihe  $p_{ikl\dots}, q_{ikl\dots}, \dots$  ist, können wir jetzt diese Reihen symbolisch zerlegen:  $p_{ikl\dots} = p_i p_k p_l \dots$ . Nun wählen wir, den  $n$  Indizes  $1, 2, \dots, n$  entsprechend,  $n$  Reihen von Raumkoordinaten  $u', v', \dots, w'$  und ersetzen in  $R'$  jedes  $p_i$  durch  $(pu')$ , jedes  $q_i$  durch  $(qu')$ ,  $\dots$  jedes  $p_n$  durch  $(pw')$  u.s.w. Aus  $R'$  entsteht auf diese Weise eine Komitante

$$K = R'((pu'), (pv'), \dots, (qu'), (qv'), \dots) \quad (6)$$

und von  $K$  kommen wir auf  $R'$ , d.h. auf  $R$  zurück, wenn wir die Matrix  $((u' v' \dots w'))$  als Einheitsmatrix wählen.

Durch  $K \equiv 0 \{u', v', \dots, w'\}$  wird also ein invariantes Gleichungssystem gegeben, wovon eine Gleichung die ursprüngliche  $p$ -Relation  $R = 0$  darstellt. Es ist dies ein besonderer Fall des Satzes von GRAM<sup>1)</sup>, denn alle irreduziblen  $p$ -Relationen desselben Grades bilden ein invariantes Gleichungssystem.

## § 2. Die quadratischen $p$ -Relationen.

Wir führen diese Zusammenfassung der irreduziblen  $p$ -Relationen desselben Grades  $m$  durch Komitanten  $K$  für den einfachsten Fall  $m=2$  näher aus.

Ersetzen wir in (3) die ersten Faktoren durch  $p_{ikl\dots}$ , die zweiten durch  $q_{ikl\dots}$ , dann entsteht, wenn wir noch  $i_{d+1}$  an Stelle von  $k_1$  schreiben:

$$\begin{aligned} \Pi &= p_{i_1 i_2 \dots i_d} \cdot q_{i_{d+1} k_2 \dots k_d} - p_{i_{d+1} i_2 \dots i_d} \cdot q_{i_1 k_2 \dots k_d} + \dots = \left\{ \begin{array}{l} \\ \end{array} \right. \quad (7) \\ &= (\Sigma \pm p_{i_1} p_{i_2} \dots p_{i_d} q_{i_{d+1}}) q_{k_2} \dots q_{k_d}, \end{aligned}$$

oder, da die eingeklammerte Summe gleich der durch  $d!$  geteilten Determinante

$$(p p \dots p q)_{i_1 i_2 \dots i_d i_{d+1}} = (p^d q)_{i_1 i_2 \dots i_d i_{d+1}}$$

ist:

$$d! \Pi = (p^d q)_{i_1 \dots i_{d+1}} q_{k_2} \dots q_{k_d} \quad (8)$$

Hieraus finden wir die Komitante

$$K = (p^d q \pi^{n-d-1}) (q q')^{d-1} \quad (9)$$

wo  $\pi_{j_1 \dots j_{n-d-1}}$  und  $q'_{i_2 \dots i_d}$  willkürliche alternierende Tensoren bedeuten. Von  $K$  gelangt man, bis auf einen von Null verschiedenen numerischen Faktor zur ursprünglichen quadratischen  $p$ -Relation zurück, indem man alle  $\pi_{j_1 \dots j_{n-d-1}}$ , bei denen die Indexgruppe  $(j_1 \dots j_{n-d-1})$  vom Komplement von  $(i_1 \dots i_{d+1})$  verschieden ist und ebenso alle  $q'_{i_2 \dots i_d}$  wo  $(i_2 \dots i_d) \neq (k_2 \dots k_d)$  ist, gleich Null setzt.

$$K \equiv 0 \{ \pi_{ikl\dots}, q'_{rst\dots} \} \quad (10)$$

ergibt dann ein invariantes Gleichungssystem das (7) enthält, d.h. das jede quadratische  $p$ -Relation enthält.

<sup>1)</sup> Vgl. Inv. Theorie, p. 160.

Bei (9) können wir statt der  $p_{i_1 \dots i_d}$  auch die Raumkoordinaten  $p'_{j_1 \dots j_{n-d}}$  einführen:

$$K = d! (p' q) (p' \pi)^{n-d-1} (q \varrho')^{d-1}$$

und vermöge (8) wird jetzt:

$$\Pi = (p' q) p'_{j_1 \dots j_{n-d-1}} q_{k_2 \dots k_d} \dots \dots \dots (11)$$

Dies ist eine besonders einfache symbolische Darstellung der  $p$ -Relationen (7). Sie kann jedoch noch durch eine andere ersetzt werden, nämlich durch Gleichungen der Gestalt

$$X_2 = (p' q)^2 p'_{j_1 \dots j_{n-d-2}} q_{k_1 \dots k_{d-2}} = 0, \dots \dots \dots (12)$$

die also entstehen aus Komitanten mit einem Klammerfaktor  $(p^d q^2 \pi^{n-d-2})$ .

Gehen wir nämlich von (9) aus, dann können wir durch identisches Umformen alle  $d-1$  Reihen  $q$  von  $(q \varrho')^{d-1}$  in den Klammerfaktor  $(q p^d \pi^{n-d-1})$  hineinbringen, wodurch  $K$  in eine Summe von Komitanten der Gestalt

$$(q^d p^m \pi^{n-d-m}) (p \varrho')^{d-m} (\pi \varrho')^{m-1} \quad (m = 2, 3, \dots) \dots \dots (13)$$

verwandelt wird<sup>1)</sup>. Diese Komitanten (13) haben aber alle die Gestalt  $\Sigma X_2$ , wo  $X_2$  durch (12) gegeben ist. Somit ist jede quadratische  $p$ -Relation in der Form

$$\Pi = \Sigma X_2 \dots \dots \dots (14)$$

darstellbar. Aber es ist auch umgekehrt jedes  $X_2$  eine Summe von Ausdrücken  $\Pi$ :

$$X_2 = \Sigma \Pi \dots \dots \dots (15)$$

Dies sieht man sofort, wenn man in  $X_2 = 0$  oder in

$$(p^d q^2)_{i_1 i_2 \dots i_{d+1} k_2} q_{k_3 \dots k_d} = 0$$

die Determinante  $(q^d p^2)_{i_1 i_2 \dots i_{d+1} k_2}$  nach der letzten Zeile entwickelt, wodurch

$$\Sigma_{i_1 \dots k_2} \pm (p^d q)_{i_1 \dots i_{d+1}} q_{k_2 k_3 \dots k_d} = 0$$

entsteht.

Das Studium der höheren  $p$ -Relationen (d.h. der vom dritten und höheren Grad in den  $p_{ikl \dots}$ ) hat dann von den Komitanten (13) auszugehen. So erhält man z.B. alle  $p$ -Relationen dritten Grades, wenn man in (13) eine oder mehr Reihen  $\pi$  durch das mit  $p$  und  $q$  äquivalente  $r$  ersetzt oder analog  $\varrho'$  durch  $r'$ . Dies gibt Komitanten der beiden Typen

$$\begin{aligned} & (p^d q^m r^h \pi^{n-d-m-h}) (q \varrho')^{d-m} (r \sigma')^{d-h} \\ & (p^d q^m \pi^{n-d-m}) (q \varrho')^{d-m-h} (q r')^h (r' \sigma)^{d-h}. \end{aligned}$$

Doch ist dies bisher nur für den einfachsten Fall  $d=2$  bei beliebigem  $n$  näher ausgeführt

<sup>1)</sup> Vergl. Inv. Theorie. p. 86.

### § 3. Syzygien bei fünf binären Linearformen.

Nehmen wir jetzt  $n=5$ ,  $d=2$ . Dann sind die zehn Grössen  $p_{ik}$  Linienkoordinaten im  $G_5$ . Hier ist die Struktur der  $p$ -Relationen bekannt und von W. H. YOUNG aufgedeckt <sup>1)</sup>.

Transponieren wir die Matrix

$$\mathfrak{M} = \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & y_4 & y_5 \end{vmatrix}, \text{ so entsteht } \mathfrak{M}' = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_5 & y_5 \end{vmatrix}$$

und hier kann jedes Paar  $x_i, y_i$  als homogene Koordinaten eines Punktes  $P_i$  in einem binären Gebiete betrachtet werden. Die  $p_{ik}$  sind dann die zehn relativen projektiven Invarianten der fünf Punkte  $P_i$  und eine irreduzible  $p$ -Relation wird zu einer Syzygie zwischen diesen Invarianten. Die Struktur aller  $p$ -Relationen aufdecken heisst jetzt: alle irreduziblen Syzygien erster, zweiter, ... Art angeben, bis nach dem HILBERT'schen Satze <sup>2)</sup> die Syzygienkette abbricht.

Hiebei heisst eine Syzygie  $S=0$  irreduzibel, wenn keine Darstellung  $S=S^{(1)}+S^{(2)}+\dots$  möglich ist, wo jedes  $S^{(i)}=0$  schon für sich eine Syzygie vorstellt. Eine Syzygie  $m$ -ter Art  $S_m=0$  liegt vor, wenn

$$S_m = N_1 S_{m-1}^{(1)} + N_2 S_{m-1}^{(2)} + \dots = 0 \quad . \quad . \quad . \quad (16)$$

ist, wo die  $S_{m-1}^{(i)}$  Syzygien  $(m-1)$ -ter und die  $N_i$  Polynome der Invarianten  $p_{ik}$  sind, wobei die folgenden Bedingungen erfüllt werden:

a. Die  $N_i$  sind nicht identisch Null, wenn die Invarianten durch unabhängig Veränderliche ersetzt werden.

b. Drückt man die  $S_{m-1}$  in (16) durch  $S_{m-2}$  aus, sodass  $S_m = \sum M_\nu S_{m-2}^{(\nu)}$  entsteht, dann ist jedes  $M_\nu$  als Polynom der Invarianten betrachtet, identisch Null.

c. Für eine Syzygie erster Art  $S_1$  gilt:  $S_1 \equiv \equiv 0 \{p_{ik}\} \equiv 0 \{x_i, y_i\}$ .

d. Drückt man in einer  $S_2 = \sum N_i S_1^{(i)}$  die  $S_1$  durch die Invarianten aus, so entsteht identisch Null.

In unserem Falle sind also die irreduziblen Syzygien erster Art  $S_1$  gegeben durch die fünf quadratischen  $p$ -Relationen.

$$A'_i = (p'q')^2 q'_i = 0 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Setzen wir nun, mit noch unbestimmten  $X_i$ , für die Syzygien  $S_2$  an:

$$B = \sum X_i A'_i = (X A') = (p'q')^2 (q' X),$$

dann sind wegen  $A'_i \equiv \equiv 0 \{p_{ik}\}$  die  $X_i$  nicht konstant und da  $B=0$  aus einer Komitante entstehen muss, haben wir  $X_i = r_i (rU')$  zu setzen. Also wird:

$$B = (A' r) (r U') = (p'q')^2 (q' r) (r U').$$

<sup>1)</sup> Atti di Torino 34 (1899) p. 596—599. Vgl. auch Inv. Theorie, p. 176.

<sup>2)</sup> Vgl. Inv. Theorie, p. 171.

Hier ist  $r_i(rU') \equiv 0 \{r_{ik}\}$ , dagegen  $B \equiv 0 \{p_{ik}\}$  weil

$$(p\ q')^2 (q'\ r) r_i \equiv 0 \{p_{ik}\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Deshalb sind durch:

$$B = (A' r) r_i = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

alle irreduziblen  $S_2$  gegeben.

Bei  $S_3$  haben wir schliesslich den Ansatz

$$C = \sum U_i B_i = (A' r) (r U') \equiv 0 \{B_i\}, \quad \equiv 0 \{A'_i\},$$

also  $r_i(rU') \equiv 0 \{p_{ik}\}$ , woraus nach (18) und (19)  $U_i = q'_i (q'p)^2 = A'_i$  folgt.

Daher ist

$$C = \sum A'_i B_i = (A' B) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

die einzige Syzygie dritter Art. Hier endigt die Syzygienkette, denn  $D = M \cdot C = \sum M A'_i B_i \equiv 0 \{B_i\}$  hat nur die Lösung  $M = 0$ .

#### § 4. Syzygien bei sechs binären Linearformen.

Bedeutend komplizierter liegen die Verhältnisse bei  $n=6$ ,  $d=2$ . Hier können wir die  $\binom{6}{2} = 15$  Grössen  $p_{ik}$  als Invarianten von sechs binären Linearformen betrachten und haben dann als irreduzible Syzygien  $S_1$  die 15 quadratischen  $p$ -Relationen

$$A'_{ik} = (p\ q')^2 q'_{ik} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Eine  $S_2$  entspringt aus einer identisch verschwindenden Komitante  $K$ , die linear ist in den  $A'_{ik}$  und bei der man überdies voraussetzen kann, dass sie höchstens drei Reihen  $x, y, z$  und höchstens drei Reihen  $u', v', w'$  enthält. Sind nämlich in  $K$  z.B. sechs Reihen  $x, y, \dots, z$  vorhanden, so entwickeln wir  $K$  in eine Gordan-Capellische Reihe

$$K = K_0 + (x\ y \dots z) \cdot K_1 + (x\ y \dots z)^2 \cdot K_2 + \dots$$

und hier liefern bereits  $K_0 = 0$ ,  $K_1 = 0, \dots$  Syzygien, sodass die aus  $K = 0$  entspringenden  $S$  nicht irreduzibel sind. Sind in  $K$  fünf Reihen  $x, y, \dots, z$  vorhanden, so ermöglicht eine analoge Reihenentwicklung die Zusammenfassung  $u'_1 = (x\ y \dots z)_{23456}$  u.s.f. zu einer Reihe  $u'$ . Sind schliesslich vier Reihen  $x, y, z$  und  $t$  in  $K$  vorhanden, so kann man sie zu  $(u'v')_{12} = (xyzt)_{3456}$  u.s.f. zusammenfassen.

Um die irreduziblen  $S_2$  zu finden haben wir deshalb zuerst alle Komitanten  $K$  zu ermitteln, die linear in den  $A'_{ik} = A_{i,p,q,z}$  sind und höchstens je drei Reihen  $x, y, z$  und  $u', v', w'$  enthalten.

Zur Verfügung stehen die Reihen:

$$A', p', q', \dots, u', v', w'; A, p, q, \dots, x, y, z \quad . \quad . \quad . \quad . \quad (22)$$

Hieraus sind Faktoren erster und zweiter Art zu bilden. Von letzteren können wir absehen, da bei Klammerfaktoren mit Komplexsymbolen ( $A', p', A$  und  $p$ ) immer Umformung auf Produkte von Linearfaktoren möglich ist. Da ferner  $(Ap')$  auf  $(A'p)$  reduziert werden kann und weiters  $(pq')$  wegen



$$(p \ q') (p \ u') (q' \ x) (q' \ y) (q' \ z) = \frac{1}{4} [(u' \ x) (A' \ y) (A' \ z) + (u' \ y) (A' \ z) (A' \ x) + (u' \ z) (A' \ x) (A' \ y)] \quad (23)$$

als Reduzent zu betrachten ist, bleiben für den Aufbau von  $K$  die Linearfaktoren übrig:

$$(A' p), (A' x), (A' u'), (p' x), (p' u'), (u' x) \quad . \quad . \quad . \quad (24)$$

Hieraus ergeben sich dann:

$$\left. \begin{aligned} K_1 &= (A' p)^2, \quad K_2 = (A' p) (A' x) (p' u'), \quad K_3 = (A' x) (A' y), \\ K_4 &= (A' p) (p' u') (A' q) (q' v') \end{aligned} \right\} \quad (25)$$

wozu noch die Typen

$$K_5 = (p' u') (p' v') \quad , \quad K_6 = (u' x) \quad . \quad . \quad . \quad (26)$$

als Komitanten ohne  $A'_{ik}$  kommen.

Setzen wir:

$$J = \frac{1}{2} (p^2 q^2 r^2) = (p' q)^2 (p' r)^2 = (A' r)^2, \quad . \quad . \quad . \quad (27)$$

dann ist  $K_1 = J$  und wir haben die Beziehung:

$$(A' r) (A' x) (r' u') = (p' q')^2 (q' r) (r' u') (q' x) = \frac{1}{6} \cdot J \cdot (u' x) \quad . \quad (28)$$

Dies gibt:

$$(B' x) (B' u') = (A' p) (A' x) (p' u') - \frac{1}{6} (A' p)^2 \cdot (u' x) \equiv 0 \{x, u', p_{ik}\} \quad (29)$$

Hier ist  $B'_k B_i \equiv 0 \{A'_{ik}\}$ , aber  $\equiv 0 \{p_{ik}\}$ , d.h. wenn die  $A'_{ik}$  durch die  $p_{ik}$  ausgedrückt werden. Somit sind  $B'_k B_i = 0$  irreduzible Syzygien  $S_2$  wovon aber wegen (Verjüngung von (29)):

$$(B' B) = (A' p)^2 - (A' p)^2 \equiv 0 \{A'_{ik}\} \quad . \quad . \quad . \quad (30)$$

nur fünf von den sechs Gleichungen  $B'_i B_i = 0$  zu den irreduziblen  $S_2$  zu rechnen sind, sodass wir durch (29)  $6^2 - 1 = 35$  irreduzible  $S_2$  dargestellt erhalten:

$$B'_k B_i = (A' p) A'_k p_i - \frac{1}{6} (A' p)^2 \delta_i^k = 0 \quad . \quad . \quad . \quad (31)$$

Jetzt wollen wir noch beweisen, dass dies alle irreduziblen  $S_2$  sind.  $K$  ist linear und homogen in den Komitanten der Typen (25). Wegen (29) können wir nach Aufzählung der Syzygien (31)  $K_2$  und  $K_4$  weglassen, da jetzt der Faktor  $(A' p)$  als Reduzent zu betrachten ist. Somit ist  $K$  linear in  $K_1 = (A' p)^2$  und  $K_3 = A'_{yz}, A'_{zx}, A'_{xy}$  [Wir schreiben im Weiteren kurz

$$\begin{aligned} A'_{xy} &\text{ für } (A' x) (A' y), \\ q'_{xyz} &\text{ für } (q' x) (q' y) (q' z) \quad \text{u.s.w.}] \end{aligned}$$

mit Koeffizienten  $\varphi$  und  $\psi_i$ , die Polynome in  $K_5$  und  $K_6$  von (26) sind:

$$K = (A' p)^2 \cdot \varphi + \sum_1^3 A'_{yz} \cdot \psi_1 \equiv 0 \{p_{ik}\}.$$

Wählen wir hier  $p_{ik}$  halbspeziell, d.h.  $J = (A' p)^2 = 0$ , aber nicht alle  $A'_{ik}$  gleich Null, so folgt aus

$$A'_{yz} \cdot \psi_1 + A'_{zx} \cdot \psi_2 + A'_{xy} \cdot \psi_3 \equiv 0$$

für  $z_i = p_i (p' \sigma')$ , dass  $\psi_3 \equiv 0$  wird, wo  $\psi_3$  ein Polynom ist mit den

Argumenten  $K_5 = p_{v'w'}, p_{w'u'}, p_{u'v'}$  und  $K_6 = (u'x), (u'y), (u'p)(p\sigma')$  u.s.w. Hieraus folgt aber  $\psi_3 = 0 \{p_{ik}\}$ ; also ist auch  $\varphi \equiv \{p_{ik}\}$  und daher  $K = 0 \{A'_{ik}\}$ , es liegt also keine  $S_2$  vor und durch (29) sind alle  $S_2$  aufgezählt.

### § 5. Die Syzygien dritter Art $S_3$ .

Hier müssen wir die Komitanten  $L$  ermitteln, die linear sind in den durch (29) gegebenen  $B'_k B_i$ .  $L$  enthält also die Reihen

$B', A', A_1, \dots, p', q', \dots, u', v', w'; B, A, A_1, \dots, p, q, \dots, x, y, z.$  (32)  
wo  $A_1, A_2, \dots$  äquivalent sind mit  $A'$ . Hieraus ergeben sich die Faktoren

$$\left. \begin{array}{cccc} (B' B), & (B' A), & (B' p), & (B' x) \\ (A' B), & (A' A_1), & (A' p), & (A' x) \\ (p' B), & (p' A), & (p' q), & (p' x) \\ (u' B), & (u' A), & (u' p), & (u' x) \end{array} \right\} \dots \dots \dots (33)$$

$(p'q')$  ist ein Reduzent;  $(p' A)$  führt auf  $(pA')$  und gibt nach (28) Reduktion auf  $(A'p)^2$ .  $(B' B)$  ist nach (30)  $\equiv 0 \{A'_{ik}\}$ , ist aber trotzdem keine  $S_3$ , da die Koeffizienten der  $B'_k B_i$  konstant sind. Auch der Faktor  $(A' A_1)$  ist ein Reduzent; es ist nämlich

$$(A' A_1)(A' x) = \frac{1}{2} (p^2 q^2 A_1 x)$$

und bringt man hier alle drei weiteren Reihen  $A_1$  in den Klammerfaktor, so entstehen Glieder mit

$$(A'^4 p x) = 24 (A' p)(A' x),$$

was Reduktion auf  $(A' p)^2$  ergibt.

Aus den übrigen Faktoren von (33) erhält man die folgenden Komitanten:

$$\left. \begin{array}{ll} L_1 = (B' x)(B u') & L_4 = (B' q)(q u')(B v') \\ L_2 = (B' x)(B q') q'_{xyz} & L_5 = (B' q)(q u')(B r') r'_{xyz} \\ L_3 = (B' x)(B A')(A' y) & L_6 = (B' q)(q u')(B A')(A' x) \\ L_7 = (B' A) A_{u'v'w'}(B u') & \\ L_8 = (B' A) A_{u'v'w'}(B q') q'_{xyz} & \\ L_9 = (B' A) A_{u'v'w'}(B A')(A' x) & \end{array} \right\} \dots (34)$$

Dazu kommen noch die Komitanten ohne  $B'_k B_i$ , die schon bei (25) und (26) aufgezählt wurden, nämlich:

$$(A' p)^2, \quad A'_{xy}, \quad p_{u'v'}, \quad (u' x) \dots \dots \dots (35)$$

Von (34) sind  $L_7, L_8$  und  $L_9$  reduzibel auf  $L_4, L_5$  und  $L_6$ , wenn wir in  $(B' A) A_{u'v'w'}$   $A^4$  durch  $p^2 q^2$  ausdrücken.

Es sei nun  $L = 0$  eine  $S_3$ .  $L$  ist linear in den Komitanten  $L_1$  bis  $L_6$  mit Koeffizienten, die Polynome der Komitanten (35) sind. Da  $L \equiv 0 \{A'_{ik}\}$  sein muss wenn wir die  $B'_k B_i$  nach (29) durch die  $A'_{ik}$  ausdrücken, müssen wir

zuerst die  $L_i$  von (34) linear in den  $A'_{ik}$  schreiben. Wir wollen dann  $M'_{ik}$  statt derjenigen  $A'_{ik}$  schreiben, die von den  $B'_k B_i$  stammen. Aus diese Weise erhalten wir:

$$\begin{aligned}
 L_1 &= (M'p)(M'x)(pu') - \frac{1}{6}(M'p) \cdot (u'x) \\
 L_2 &= (M'p)(M'x)(pq')q'_{xyz} = \frac{1}{4}[(M'y)(M'x) \cdot A'_{xz} - (M'z)(M'x) \cdot A'_{xy}] \\
 L_3 &= (M'p)(M'x)(pA')(A'y) - \frac{1}{6}(M'p)^2 \cdot A'_{xy} = \frac{1}{6}J M'_{xy} - \frac{1}{6}(M'p)^2 \cdot A'_{xy} \\
 L_4 &= (M'p)(M'q)(pv')(qu') - \frac{1}{6}(M'p)^2 \cdot (qv')(qu') \\
 L_5 &= (M'p)(M'q)(qu')(pr')r'_{xyz} - \frac{1}{6}(M'p)^2 \cdot (qr')(qu')r'_{xyz} = \\
 &= \frac{1}{4}[(M'q)(M'x)(qu') \cdot A'_{yz} + (M'q)(M'y)(qu') \cdot A'_{zx} + \\
 &\quad + (M'q)(M'z)(qu') \cdot A'_{xy}] - \frac{1}{24}(M'p)^2 [(u'x)A'_{yz} + \\
 &\quad + (u'y)A'_{zx} + (u'z)A'_{xy}] \\
 L_6 &= (M'p)(M'q)(pA')(qu')(A'x) - \frac{1}{6}(M'p)^2 \cdot (A'q)(qu')(A'x) = \\
 &= \frac{1}{6}J[(M'q)(M'x)(qu') - \frac{1}{6}(M'p)^2 \cdot (u'x)].
 \end{aligned} \tag{36}$$

In diesen Formeln ist die Reduktion ausgeführt, wenn ein Faktor  $(pq')$  oder ein Faktor  $(pA')$  vorhanden war.

Nach (25) haben wir also:

$$\begin{aligned}
 L_1(x, u') &= K_2(x, u') - \frac{1}{6}K_1 \cdot (u'x) \\
 L_2(x, xyz) &= \frac{1}{4}[K_3(x, z) \cdot A'_{xy} - K_3(x, y) \cdot A'_{xz}] \\
 L_3(x, y) &= \frac{1}{6}J \cdot K_3(x, y) - \frac{1}{6}K_1 \cdot A'_{xy} \\
 L_4(u', v') &= K_4(v', u') - \frac{1}{6}K_1 \cdot p_{v'u'} \\
 L_5(u', xyz) &= \frac{1}{4}\{[K_2(x, u') - \frac{1}{6}K_1 \cdot (u'x)]A'_{yz} + \\
 &\quad + [K_2(y, u') - \frac{1}{6}K_1 \cdot (u'y)] \cdot A'_{zx} + [K_2(z, u') - \frac{1}{6}K_1 \cdot (u'z)] \cdot A'_{xy}\} \\
 L_6(x, u') &= \frac{1}{6}J[K_2(x, u') - \frac{1}{6}K_1 \cdot (u'x)].
 \end{aligned} \tag{37}$$

Hieraus folgen:

$$L_6 - \frac{1}{6}J L_1 \equiv 0 \{M'_{ik}\} \dots \dots \dots \tag{38}$$

$$L_5 - \frac{1}{4}[L_1(x, u') \cdot A'_{yz} + L_1(y, u') \cdot A'_{zx} + L_1(z, u') \cdot A'_{xy}] \equiv 0 \{M'_{ik}\}. \tag{39}$$

Wir können also jedes  $L_5$  und  $L_6$  durch  $L_1$  ausdrücken und haben die  $S_3$  aufzuzählen, die entspringen aus:

$$(C'x)(Cu') = (B'q)(qu')(BA')(A'x) - \frac{1}{6}J(B'x)(Bu') = 0. \tag{40}$$

$$\begin{aligned}
 (Du') \triangle'_{xyz} &= (B'q)(qu')(Br')r'_{xyz} - \frac{1}{4}[(B'x)(Bu') \cdot A'_{yz} + \\
 &\quad + (B'y)(Bu') \cdot A'_{zx} + (B'z)(Bu') \cdot A'_{xy}] = 0.
 \end{aligned} \tag{41}$$

Jedes weitere  $L$  ist jetzt linear und homogen in  $L_1$  bis  $L_4$ . Bei  $L_1$  haben wir entsprechend den sechs Reihen  $x, y, z, u', v', w'$ , neun Komitanten  $L_1(x, u')$ ,  $L_1(y, u')$ , ... Bei  $L_2$  haben wir drei Komitanten, die nach (37) verknüpft sind durch die Beziehung:

$$L_2(x, xyz) \cdot A'_{yz} + L_2(y, xyz) \cdot A'_{zx} + L_2(z, xyz) \cdot A'_{xy} \equiv 0 \{K_3\}. \tag{42}$$

Hier ergeben sich also wieder  $S_3$ :

$$\left. \begin{aligned} (\alpha'x)^2 (\beta'y)^2 (\gamma'z)^2 &= (B'x) (Bq') q'_{xyz} \cdot A'_{yz} + \\ &+ (B'y) (Bq') q'_{xyz} \cdot A'_{zx} + (B'z) (Bq') q'_{xyz} \cdot A'_{xy} = 0. \end{aligned} \right\} \quad (43)$$

Bei  $L_3(x, y)$  haben wir nur drei Komitanten, da wegen  $K_3(x, y) = M'_{xy}$   $K_3(x, x) = 0$  wird und also auch  $L_3(x, x) \equiv 0 \{M'_{ik}\}$  ist, was wieder Syzygien dritter Art liefert, entspringend aus:

$$(R'x)^2 = L_3(x, x) = (B'x) (BA') (A'x) = 0 \quad \dots \quad (44)$$

Ebenso haben wir bei  $L_4(u', v')$  nur drei Komitanten; denn auch hier ist wegen  $K_4(u', v') = (M'p) (pu') (M'q) (qv') = -K_4(v', u')$ :

$$(Ru')^2 = L_4(u', u') = (B'q) (qu') (Bu') = 0 \quad \dots \quad (45)$$

Machen wir jetzt den allgemeinen Ansatz

$$L = \sum_9 L_1(x, u') \cdot \alpha_{11} + \sum_3 L_2(x, xyz) \cdot \beta_1 + \sum_3 L_3(x, y) \cdot \gamma_3 + \sum_3 L_4(u', v') \cdot \delta_3, \quad (46)$$

dann muss dies  $\equiv 0 \{M'_{ik}\}$  sein, wenn wir die  $L_i$  nach (37) durch die  $K_i$  ausdrücken und setzen:

$$\left. \begin{aligned} K_1 &= (M'p)^2, \quad K_2 = (M'p) (M'x) (pu'), \quad K_3 = M'_{xy}, \\ K_4 &= (M'p) (pu') (M'q) (qv'). \end{aligned} \right\} \quad (47)$$

Die Koeffizienten  $\alpha_{ik}$ ,  $\beta_i$ ,  $\gamma_i$  und  $\delta_i$  in (46) sind Polynome der Typen:

$$J = (A'p)^2, \quad A'_{xy}, \quad p_{u'v'}, \quad (u'x) \quad \dots \quad (48)$$

Bei  $K_2$  in (47) haben wir 9, bei  $K_3$  und  $K_4$  je drei Komitanten. (46) geht daher über in:

$$\left. \begin{aligned} (M'p)^2 \cdot \Phi + \sum_9 (M'p) (M'x) (pu') \cdot \alpha_{11} + \\ + \sum_3 M'_{xy} \cdot \chi_3 - \sum_3 (M'p) (pu') (M'q) (qv') \cdot \delta_3 \equiv 0 \{M'_{ik}\}. \end{aligned} \right\} \quad (49)$$

Dabei ist nach (46):

$$\Phi = -\frac{1}{6} \sum_9 (u'x) \cdot \alpha_{11} - \frac{1}{6} \sum_3 A'_{xy} \cdot \gamma_3 + \frac{1}{6} \sum_3 p_{u'v'} \cdot \delta_3 \quad \dots \quad (50)$$

(49) muss gelten für alle  $M'_{ik}$ . Setzen wir  $M'_{ik} = (\sigma' \tau')_{ik}$ , dann erhalten wir:

$$\left. \begin{aligned} 2p_{\sigma'\tau'} \cdot \Phi + \sum_9 [p_{\sigma'u'}(\tau'x) - p_{\tau'u'}(\sigma'x)] \cdot \alpha_{11} + \sum_3 [(\sigma'x)(\tau'y) - (\sigma'y)(\tau'x)] \cdot \chi_3 - \\ - \sum_3 [p_{\sigma'u'} \cdot q_{\tau'v'} - p_{\tau'u'} \cdot q_{\sigma'v'}] \cdot \delta_3 \equiv 0 \{\sigma', \tau'\}. \end{aligned} \right\} \quad (51)$$

Wählen wir jetzt  $\sigma'$  so, dass das lineare Gebiet  $G_5(\sigma')$  durch die 5 Punkte  $x, y, z, p_i(pu')$  und  $q_i(qv')$  geht, dass also:

$$\sigma'_1 = (x y z p q)_{23456} (pu') (qv')$$

ist und setzen gleichzeitig:

$$\tau'_1 = (x y z p q)_{23456} (pu') (qv'),$$

so ist erstens bei nicht-speziellen  $p_{ik} : (p\sigma')(p\tau') \equiv 0$  und zweitens verschwinden in (51) alle Glieder bis auf das erste. Also ist  $\Phi \equiv 0$ .



Wählen wir dann  $\sigma'$  wieder so wie eben ausgeführt, hingegen  $\tau'$  so, dass das  $G_5(\tau')$  durch  $x, y, z$  und  $p_i(pu')$  geht, so ergibt sich  $\delta_3 = 0$ ; es verschwinden also auch alle  $\delta_i$ . Wählen wir dann  $\sigma'$  so, dass

$$(\sigma'x) = (\sigma'y) = (\sigma'z) = 0$$

ist, so folgt  $a_{ik} = 0$  und es bleibt:

$$\sum_3 K_3(x, y) \cdot \chi_3 = \sum_3 M'_{xy} \cdot \chi_3 = 0, \quad . \quad . \quad . \quad . \quad (52)$$

woraus leicht  $\chi_i = 0$  gefolgert wird. Statt (46) kommt also jetzt:

$$L = \sum_3 L_2(x, xy, z) \cdot \beta_1 + \sum_3 L_3(x, y) \cdot \gamma_3 = 0, \quad . \quad . \quad . \quad (53)$$

wobei wegen  $\Phi = 0$  nach (50) die Gleichungen bestehen:

$$\sum_3 A'_{xy} \cdot \gamma_3 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

$$\left. \begin{aligned} \chi_1 &= +\frac{1}{4}\beta_2 A'_{xy} - \frac{1}{4}\beta_3 A'_{xx} + \frac{1}{6}J \cdot \gamma_1 = 0 \\ \chi_2 &= +\frac{1}{4}\beta_3 A'_{yz} - \frac{1}{4}\beta_1 A'_{xy} + \frac{1}{6}J \cdot \gamma_2 = 0 \\ \chi_3 &= +\frac{1}{4}\beta_1 A'_{xx} - \frac{1}{4}\beta_2 A'_{yz} + \frac{1}{6}J \cdot \gamma_3 = 0 \end{aligned} \right\} \quad . \quad . \quad . \quad (55)$$

Sind in (55) alle  $\beta_i = 0$ , so folgt  $\gamma_i = 0$ , wir erhalten keine  $S_3$ . Ist  $\beta_2 = \beta_3 = 0$ ,  $\beta_1 \neq 0$ , so folgt  $\gamma_1 = 0$  und nach (55):

$$\frac{1}{4}\beta_1 \cdot A'_{xy} = \frac{1}{6}J \cdot \gamma_2 \quad , \quad \frac{1}{4}\beta_1 A'_{xx} = -\frac{1}{6}J \cdot \gamma_3$$

Aus (53) ergibt sich dann die Komitante

$$J \cdot L_2(x, xy, z) - \frac{3}{2}[L_3(x, z) A'_{xy} - L_3(x, y) A'_{xz}] \equiv 0 \{M'_{ik}\}, \quad . \quad (56)$$

aus welcher wieder Syzygien dritter Art entstehen:

$$(E'x)^2 (F'y) (F'z) = J \cdot (B'x) (B'q') q'_{xyz} - \frac{3}{2} \left[ (B'x) (BA') (A'z) \cdot A'_{xy} - \right. \\ \left. - (B'x) (BA') (A'y) \cdot A'_{xz} \right] \quad \left. \vphantom{\frac{3}{2}} \right\} \quad (57)$$

Sind in (55) schliesslich alle  $\beta_i \neq 0$ , so folgt  $\gamma_i = 0$ , da die Koeffizienten-determinante der  $\beta_i$  in diesen Gleichungen verschwindet. Es bleiben dann drei lineare und homogene Gleichungen für die  $\beta_i$ , deren Lösung auf die schon aufgezählte Beziehung (42) führt.

## § 6. Die irreduziblen $S_3$ .

Nach (44) haben wir

$$(R'x) (R'y) = \frac{1}{2} [(B'x) (BA') (A'y) + (B'y) (BA') (A'x)];$$

ersetzen wir hier  $y_i$  durch  $p_i(pu')$ , so entsteht:

$$\begin{aligned} (R'x) (R'p) (pu') &= \frac{1}{2} [(B'x) (BA') (A'p) (pu') + (B'p) (pu') (BA') (A'x)] = \\ &= \frac{1}{2} [-\frac{1}{6}J \cdot (B'x) (Bu') + L_6] = \frac{1}{2} [L_6 - \frac{1}{6}J L_1], \end{aligned}$$

also nach (38) und (40):

$$(C'x)(Cu') = 2(R'x)(R'p)(pu') \dots \dots \dots (58)$$

und dies gilt identisch in allen  $B'_k B'_l$ . Die durch  $C'_k C'_l = 0$  dargestellten Syzygien dritter Art sind also auf  $R'_{ik} = 0$  reduzierbar.

Auf analoge Weise bekommen wir bei (45) aus

$$(Ru')(Rv') = \frac{1}{2} [(B'q)(qu')(Bv') + (B'q)(qv')(Bu')]$$

wenn wir  $v' = p' p'_{xyz}$  setzen:

$$(Ru')(Rp') p'_{xyz} = \frac{1}{2} [(B'q)(Bu')(qp') p'_{xyz} + (B'q)(Bp')(qu') p'_{xyz}],$$

also nach (23), (34), (39) und (41):

$$(Du') \Delta'_{xyz} = 2(Ru')(Rp') p'_{xyz} \dots \dots \dots (59)$$

d. h. auch die Syzygien  $D_i \Delta'_{klm} = 0$  sind reduzibel.

Ersetzen wir weiters in  $(C'x)(Cu')$  die  $u'$  durch  $p' p'_{xyz}$ , so finden wir nach (40):

$$(C'x)(Cp') p'_{xyz} = (B'q)(BA')(A'x)(qp') p'_{xyz} - \frac{1}{6} J \cdot (B'x)(Bp') p'_{xyz}$$

und hier führt die Umformung von  $(qp')$  nach (57) zur Beziehung

$$\left. \begin{aligned} \frac{1}{6} (E'x)^2 F'_{yz} &= -(C'x)(Cp') p'_{xyz} - \frac{1}{2} (R'x)^2 \cdot A'_{yz} - \\ &\quad - \frac{1}{2} (R'x)(R'y) \cdot A'_{zx} - \frac{1}{2} (R'x)(R'z) \cdot A'_{xy}, \end{aligned} \right\} (60)$$

d. h. nach (58):

$$(E'x)^2 F'_{yz} = \frac{3}{2} (R'x)^2 \cdot A'_{yz} \dots \dots \dots (61)$$

Es sind also auch alle Syzygien  $E'_{ik} F'_{lm} = 0$  reduzibel.

Ersetzen wir schliesslich in  $(Du') \Delta'_{xyz}$  die  $u'$  durch  $p' p'_{xyz}$ , so wird nach (41):

$$(Dp') p'_{xyz} \Delta'_{xyz} = (B'q)(qp') p'_{xyz} (Br') r'_{xyz} - \frac{1}{4} [(B'x)(Bp') p'_{xyz} \cdot A'_{yz} + \dots],$$

also nach (42) und (43), wenn wir im ersten Gliede rechts  $(qp')$  nach (23) umformen:

$$(\alpha'x)^2 (\beta'y)^2 (\gamma'z)^2 = -2(Dp') p'_{xyz} \Delta'_{xyz} \dots \dots \dots (62)$$

Nach (59) ist also:

$$(\alpha'x)^2 (\beta'y)^2 (\gamma'z)^2 = -4(Rp') p'_{xyz} (Rq') q'_{xyz} \dots \dots \dots (63)$$

wodurch auch die Syzygien  $\alpha'_{ik} \beta'_{lm} \gamma'_{rs} = 0$  reduziert sind.

Es bleiben somit die 21 irreduziblen  $S_3$ , die aus  $(R'x)^2 \equiv 0$  entstehen:

$$R'_{ik} = \frac{1}{2} [B'_i (BA') A'_k + B'_k (BA') A'_i] \equiv 0 \{B'_k B'_l\} \equiv 0 \{A'_{ik}\} \quad (64)$$

und die 21 irreduziblen  $S_3$ , die sich aus  $(Ru')^2 \equiv 0$  ergeben:

$$R_{ik} = \frac{1}{2} [(B'q) q_i B_k + (B'q) q_k B_i] \equiv 0 \{B'_k B'_l\} \equiv 0 \{A'_{ik}\} \quad (65)$$

## § 8. Die Syzygien vierter Art.

Es sei  $T = 0$  eine  $S_4$ .  $T$  ist linear in den  $R'_{ik}$  und  $R'_{lm}$  und verschwindet identisch, wenn wir diese nach (64) und (65) durch die  $B'_k B'_l$  ausdrücken.

Die Typen von linear nicht-reduziblen Komitanten mit  $R_{ik}$  und  $R'_{ik}$  sind, ausgedrückt durch die  $L_i$  von (34):

$$\begin{aligned}
 R_1 &= 2(Ru')(Rv') = L_4(u', v') + L_4(v', u') \\
 R_2 &= 2(Ru')(Rp') p'_{xyz} = L_5 - \frac{1}{4}[L_1(x, u') \cdot A'_{yz} + L_1(y, u') \cdot A'_{zx} + \\
 &\quad + L_1(z, u') \cdot A'_{xy}] \\
 R_3 &= 2(Ru')(RA')(A'x) = L_6 - \frac{1}{6}J \cdot L_1(x, u') \\
 R_4 &= 2(RA')(A'x)(Rq') q'_{xyz} = -\frac{1}{6}J L_2(x, xyz) - \frac{1}{4}[L_3(x, x) \cdot \\
 &\quad \cdot A'_{yz} + L_3(y, x) \cdot A'_{zx} + L_3(z, x) \cdot A'_{xy}] \\
 R_5 &= 2(RA')(A'x)(RB')(B'y) = -\frac{1}{6}J[L_3(x, y) + L_3(y, x)] \\
 R_6 &= 2(Rp')(p'_{xyz}(Rq) q'_{xyz} = \\
 &\quad = -\frac{1}{2}[L_2(x, xyz) \cdot A'_{yz} + L_2(y, xyz) \cdot A'_{zx} + L_2(z, xyz) \cdot A'_{xy}] \\
 R'_1 &= 2(R'x)(R'y) = L_3(x, y) + L_3(y, x) \\
 R'_2 &= 2(R'x)(R'p)(pu') = L_6(x, u') - \frac{1}{6}J \cdot L_1(x, u') \\
 R'_3 &= 2(R'p)(pu')(R'q)(qv') = -\frac{1}{6}J[L_4(u', v') + L_4(v', u')]
 \end{aligned} \tag{66}$$

Aus der letzten der Gleichungen (67) und der ersten von (66) ergibt sich:

$$\begin{aligned}
 T_1 &= (Tu')(Tv') = R'_3(u', v') + \frac{1}{6}J \cdot R_1(u', v') = \\
 &= (R'p)(pu')(R'q)(qv') + \frac{1}{6}J(Ru')(Rv') \equiv 0\{R_{ik}, R'_{rm}\} \equiv 0\{B'_k B_i\}.
 \end{aligned} \tag{68}$$

woraus 21  $S_4$  entspringen. Wir wollen beweisen, dass dies alle irreduziblen Syzygien vierter Art sind.

Eine  $S_4$   $T=0$  entsteht durch Elimination der  $L_i$  aus den Gleichungen (66) und (67). Die Elimination von  $L_4$  gibt (68), die von  $L_6$  gibt:

$$T_3 = (Tu')(TA')(A'x) = -\frac{1}{6}J \cdot R'_2(x, u') + \frac{1}{6}J \cdot R_3(x, u') \equiv 0\{B'_k B_i\} \tag{69}$$

Die Elimination von  $L_3$  gibt:

$$T_5 = (TA')(A'x)(TB')(B'y) = \frac{1}{6}J[R_5(x, y) + \frac{1}{6}J R'_1(x, y)] \equiv 0\{B'_k B_i\}. \tag{70}$$

Setzen wir ferner in (68)  $v' = p' p_{xyz}$ , so entsteht:

$$\begin{aligned}
 4(Tu')(Tp') p'_{xyz} &= \frac{2}{3}J(Ru')(Rp') p'_{xyz} - \\
 &- [(R'x)(R'p)(pu') \cdot A'_{yz} + (R'y)(R'p)(pu') \cdot A'_{zx} + (R'z)(R'p)(pu') \cdot A'_{xy}].
 \end{aligned} \tag{71}$$

Hier kann man rechter Hand nach (69) und (67)<sub>2</sub> die  $R'$  eliminieren und erhält:

$$\begin{aligned}
 4J(Tu')(Tp') p'_{xyz} - 6 \sum (Tu')(TA')(A'x) \cdot A'_{yz} = \\
 = \frac{2}{3}J R_2(u', xyz) - [R_3(x, u') \cdot A'_{yz} + R_3(y, u') \cdot A'_{zx} + \\
 + R_3(z, u') \cdot A'_{xy}] \equiv 0\{B'_k B_i\},
 \end{aligned} \tag{72}$$

woraus sich für

$$\begin{aligned}
 u' = A'(A'x) \text{ bzw. für } u' = q' q'_{xyz} \text{ ergeben:} \\
 4J(TA')(A'x)(Tp') p'_{xyz} - 6 \sum (TA')(A'x)(TB')(B'y) \cdot A'_{yz} = \\
 = \frac{2}{3}J R_4(x, xyz) - [R_5(x, x) \cdot A'_{yz} + R_5(x, y) \cdot A'_{zx} + \\
 + R_5(x, z) \cdot A'_{xy}] \equiv 0\{B'_k B_i\}
 \end{aligned} \tag{73}$$

$$\left. \begin{aligned} 4 J(Tp') p'_{xyz} (Tq') q'_{xyz} - 6 \Sigma (TA') (A'x) (Tp') p'_{xyz} \cdot A'_{yz} = \\ = \frac{8}{3} J R_6 - [R_4(x, xy z) \cdot A'_{yz} + R_4(y, xy z) \cdot A'_{zx} + \\ + R_4(z, xy z) \cdot A'_{xy}] \equiv 0 \{B'_k B_i\}. \end{aligned} \right\} \quad (74)$$

Alle diese identisch in den  $B'_k B_i$  geltenden Beziehungen sind  $S_4$  die auf  $T_1$  reduzibel sind.

Sei nun  $T=0$  eine  $S_4$ .  $T$  ist linear in den  $R_i$  und  $R'_k$  von (66) und (67). Multiplizieren wir  $T$  mit  $J^2$ , so ist jedes  $R'_3$  nach (68) durch  $R_1$ , jedes  $R'_2$  nach (69) durch  $R_3$ , jedes  $JR'_1$  nach (70) durch  $R_5$  ausdrückbar, sodass in  $T$  keine  $R'_i$  mehr vorkommen.

Jetzt können wir auf dieselbe Weise vermöge (72), (73) und (74)  $R_2$ ,  $R_4$  und  $R_6$  eliminieren. Dies gibt schliesslich für  $T$  den Ansatz:

$$J^2 T = \sum_1^6 R_1(v', w') \cdot \alpha_{23} + \sum_1^9 R_3(u', x) \cdot \beta_{11} + \sum_1^6 R_5(y, z) \cdot \gamma_{23} \equiv 0 \{B'_k B_i\} \quad (75)$$

Hier sind die  $\alpha_{ik}$ ,  $\beta_{ik}$  und  $\gamma_{ik}$  Polynome in den Komitanten der Typen

$$J = (A'p)^2, \quad A'_{xy}, \quad p_{u'v'}, \quad (u'x).$$

Aus (75) folgt nun, dass alle  $\alpha_{ik}$ ,  $\beta_{ik}$  und  $\gamma_{ik}$  Null sein müssen, d.h. dass auch  $T \equiv 0 \{R_{ik}\}$  gelten muss, d.h. (68) gibt die einzigen  $S_4$ .

Das Verschwinden der Koeffizienten in (75) beweist man wie folgt. Wir wählen  $B'_k B_i$  so, dass  $B'$  und  $B$  Grössenreihen sind:

$$B'_1 = (xyzpq)_{23456} (pu') (qv') \quad , \quad B_1 = (u'v'w'\sigma'\tau')_{23456}.$$

Dann haben wir:

$$(B'x) = 0, \quad (B'y) = 0, \quad (B'z) = 0, \quad (B'p) (pu') = 0, \quad (B'q) (qv') = 0$$

und

$$(Bu') = 0, \quad (Bv') = 0, \quad (Bw') = 0.$$

Bei diesen Annahmen reduziert sich (75) auf

$$(B'q) (qw') \cdot [(BA') (A'x) \cdot \beta_{13} + (BA') (A'y) \cdot \beta_{23} + (BA') (A'z) \cdot \beta_{33}] \equiv 0,$$

woraus wegen

$$(B'q) (qw') = (xyzpqr) (pu') (pv') (rw') \equiv 0$$

$\beta_{ik} = 0$  folgt. Daher kommt statt (75):

$$\begin{aligned} T = \sum_1^6 [(B'p) (pv') (Bw') + (B'p) (pw') (Bv')] \cdot \alpha_{23} - \\ - \frac{1}{6} J \cdot \sum_1^6 [(B'y) (BA') (A'z) + (B'z) (BA') (A'y)] \equiv 0 \{B'_k B_i\} \end{aligned}$$

Wählt man hier  $B'$  so, dass  $(B'x) = 0$ ,  $(B'y) = 0$  und  $B$  so, dass  $(Bu') = 0$ ,  $(Bv') = 0$  und  $(Bw') = 0$  ist, so folgt  $\gamma_{ik} = 0$  und analog finden wir, dass auch  $\alpha_{ik} = 0$  sein muss.

Wir haben somit 21 irreduzible  $S_4$ , die aus (68) entspringen:

$$T_{ik} = (R'p) p_i (R'q) q_k + \frac{1}{6} J \cdot R_{ik} \equiv 0 \{R'_{ik}, R_{lm}\}, \equiv 0 \{B'_k B_i\}$$

## § 9. Das Abbrechen der Syzygienkette.

Die in den  $T_{ik}$  linearen, linear-irreduziblen Komitanten sind analog zu (66) gegeben durch:

$$\left. \begin{aligned} T_1 &= (Tu')(Tv') = R'_3(u', v') + \frac{1}{6} JR_1(u', v') \\ T_2 &= (Tu')(Tp') p'_{xyz} = -\frac{1}{4} [R'_2(x, u') \cdot A'_{yz} + \dots] + \frac{1}{6} JR_2(u', xyz) \\ T_3 &= (Tu')(TA')(A'x) = -\frac{1}{6} JR'_2(x, u') + \frac{1}{6} JR_3(x, u') \\ T_4 &= (TA')(A'x)(Tp') p'_{xyz} = \frac{1}{24} J [R'_1(x, x) \cdot A'_{yz} + \dots] + \frac{1}{6} JR_4(x, xyz) \\ T_5 &= (TA')(A'x)(TB')(B'y) = \frac{1}{36} J^2 \cdot R'_1(x, y) + \frac{1}{6} JR_5(x, y) \\ T_6 &= (Tp') p'_{xyz} (Tq') q'_{xyz} = \frac{1}{16} \Sigma R'_1(x, x) \cdot A'_{yz} A'_{yz} + \frac{1}{6} J \cdot R_6. \end{aligned} \right\} \quad (76)$$

Es sei  $S=0$  eine  $S_5$ . Dann ist  $S$  linear in den Komitanten (76). Wir wollen zeigen, dass man  $J^2 \cdot S$  durch  $T_1, T_3$  und  $T_5$  allein ausdrücken kann. Hiezu ist nachzuweisen, dass die Gleichungen (72), (73) und (74) des vorigen § nicht nur für beliebige  $B'_k B_i$ , sondern auch bei willkürlichen  $R_{jk}$  gelten. Dieser Nachweis kann wie folgt erbracht werden.

Formt man  $(A'^2 B'^2 C'^2)(p' t)$  um, so entsteht:

$$(A'^2 B'^2 C'^2)(p' t) p'_{xyz} = -6 (p' A' B'^2 C'^2) p'_{xyz} (A' t) \quad (77)$$

Da aber, wie leicht nachzurechnen:

$$(A'^2 B'^2 C'^2) = \frac{1}{3} J^2 \quad (78)$$

$$(p' A' B'^2 C'^2) p'_{xyz} (A' t) = \frac{4}{3} J [A'_{xt} A'_{yz} + A'_{yt} A'_{zx} + A'_{zt} A'_{xy}] \quad (79)$$

ist, wird aus (77), wenn wir  $t=R$  setzen und mit  $(Ru')$  multiplizieren:

$$\frac{4}{3} J \cdot (Ru') (Rp') p'_{xyz} = \Sigma (Ru') (RA') (A'x) \cdot A'_{yz} \quad (80)$$

was Gleichung (72) mit beliebigem  $R_{ik}$  ist.

Wir können daher auch  $T_{ik}$  statt  $R_{ik}$  in (80) schreiben und dann in  $J^2 S$  mit Hilfe der zu (72), (73) und (74) analogen Gleichungen jedes  $JT_2$ ,  $JT_4$  und  $JT_6$  durch  $T_3$  und  $T_5$  ausdrücken. Es entsteht so:

$$\left. \begin{aligned} J^2 S &= \sum_1^6 T_{v'w'} \cdot \alpha_{23} + \sum_1^9 (Tu')(TA')(A'x) \cdot \beta_{11} + \\ &\quad + \sum_1^6 (TA')(A'y)(TB')(B'z) \cdot \gamma_{23} \end{aligned} \right\} \quad (81)$$

und dies muss  $\equiv 0 \{R'_{ik}, R_{lm}\}$  sein. Nach den Gleichungen (76) folgt dann, dass auch

$$\begin{aligned} \sum_1^6 R_{v'w'} \cdot \alpha_{23} + \sum_1^9 (Ru')(RA')(A'x) \cdot \beta_{11} + \\ + \sum_1^6 (RA')(A'y)(RB')(B'z) \cdot \gamma_{23} \equiv 0 \{R_{ik}\} \end{aligned}$$

sein muss. Hieraus folgt aber nach dem vorigen § das Verschwinden aller  $\alpha_{ik}$ ,  $\beta_{ik}$  und  $\gamma_{ik}$ , d.h.  $S \equiv 0 \{T_{ik}\}$ . Es gibt also keine  $S_5$  mehr.



Wir stellen schliesslich noch die erhaltenen Syzygien übersichtlich zusammen, wobei wir von der Schreibweise mit hoch- und tiefstehenden Indizes Gebrauch machen, indem wir z. B.  $p^{iklm}$  statt  $p'_{iklm}$  schreiben :

15 irred. Syzygien erster Art  $A^{ik} = p^{ik\lambda\mu} p_{\lambda\mu} = 0$

35    "    "    zweiter    "     $B_i^k = A^{\lambda k} p_{\lambda i} - \frac{1}{6} A^{\lambda\mu} p_{\lambda\mu} \delta_i^k = 0$

42    "    "    dritter    "     $\{R^{ik} = \frac{1}{2} [B_\lambda^i A^{\lambda k} + B_\lambda^k A^{\lambda i}] = 0$

42    "    "    dritter    "     $\{R_{ik} = \frac{1}{2} [B_i^\lambda p_{\lambda k} + B_k^\lambda p_{\lambda i}] = 0$

21    "    "    vierter    "     $T_{ik} = R^{\lambda\mu} p_{\lambda i} p_{\mu k} + \frac{1}{6} J \cdot R_{ik} = 0$

**Physiology.** — *The action of veratrin on skeletal muscles. At the same time an application of our knowledge of the segmentation of the frog's M. gastrocnemius on the problem of the refractory stage of skeletal muscles after veratrin poisoning. By S. DE BOER. (Communicated by Prof. R. MAGNUS.)*

(Communicated at the meeting of January 29, 1927).

I performed this investigation in consequence of Dr. QUERIDO's dissertation (1). From my first investigation (2) I deduced an argument in favour of the theory of BOTTAZZI (3), who ascribes the contracture of striped muscles poisoned with veratrin to a slow contraction of the sarcoplasm. At present I should rather use the expression *tonus substratum* instead of sarcoplasm to indicate that the site of the muscle-tonus and the contractures is not yet determined. As for the rest I maintain the conclusion which I drew at that time. I discriminate therefore the *tonus substratum* from the *substratum* in which the twitches occur.

To prevent confusion I wish to point out, that I shall continue to give the determinations which were used formerly. I oppose the contractures to the tetanic contractions. Both form a steady and continuous shortening of the muscle. The mechanical effect for both is therefore steady. The tetanic contraction, however, is accompanied by intermittent electrical phenomena. Therefore the tetanic contractions consist of discontinuous processes. Such discontinuous electrical processes do not accompany the contractures. We find here a continuous deflection of the string in monophasic leading off in such a way that the tissue of the muscle in contracture is electro-negative to an injured part of the muscle or to the tendon. By muscle *tonus* I mean the tension, which is caused by a slight shortening and which is maintained reflectorily in the striped muscles. This slight shortening must be freed from the last traces of tetanus, which we conclude from the fact, that the discontinuous phenomena must be absent. I represented these questions in this way in my publications of 1913 and 1914. The introduction of new denominations cannot, in my opinion, make our insight in this matter clearer.

During my investigations of 1913 I found in the monophasic leading off a continuous deflection in the electrogram, accompanied particularly in the beginning of the contracture by discontinuous deflections. After this the curve of the electrogram became smooth, while the contracture still continued. I found also, that these discontinuous deflections were more distinct after the first stimulus. Repeating the experiment, the oscillations of the electrogram curve diminished or disappeared. These results differed very slightly if at all, from those which HOFFMANN (4) obtained after strong poisoning. HOFFMANN, however, only mentions oscillations in the beginning of the contracture, after which the electrogram curve became smooth. I hope to indicate the cause of this in my theoretical discussion.

We both found therefore discontinuous electrical phenomena, *and at the same time I found a continuous deflection in the monophasic leading off as well.*

Our conclusions however are quite different. HOFFMANN gave it as his opinion that the second shortening was a tetanic contraction. That the electrogram-curve became smooth while the maximum of shortening still continued, was not according to HOFFMANN of great importance. In any case it did not alter his conception. I have already mentioned my conception. In my opinion the oscillations of the electrogram were not of fundamental importance.

Dr. QUERIDO considers that activity of the same substratum causus both shortenings of the veratrin curve (the initial twitch and the contracture). His chief argument is that he could not see any response after an induction shock at the beginning of the contracture and ordinarily not until the contracture decreased. From this experiment he concluded that the muscle was refractory. Now after my investigations on the segmentation of the M gastrocnemius, I am able to prove that this conclusion is wrong. The scheme in fig. 1 shows a veratrin curve. The so-called refractory stage should have a duration from A to B. In any case Dr. QUERIDO did not

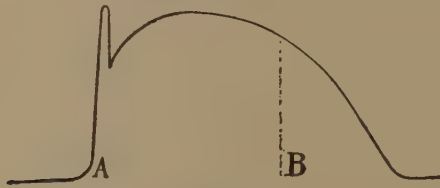


Fig. 1.

Veratricurve. Duration of the so-called refractory stage according to Dr. QUERIDO from A to B.

obtain any effect during that period after the second stimulus. I emphasize that Dr. QUERIDO arrived at this conclusion after the experiments which he performed by means of mechanical registration of the shortening curves. To make my point of view more easily understood, I mention here an observation of BIEDERMANN (5), who put one half of a muscle in water, so that this part swelled. He then stimulated the part of the muscle which had absorbed water. This swollen part did not contract, but the other part did. From this experiment BIEDERMANN concluded that the half of the muscle that had absorbed water, propagated the excitation wave but could not contract. Contractility and conductivity according to BIEDERMANN are two different properties of the skeletal muscles.

ENGELMANN (6) performed similar experiments in the heart. He put the auricles of the hearts of frogs in water until these heartsections swelled. He then stimulated the auricles which did not contract. The ventricle however contracted after an A—V interval. ENGELMANN therefore came to the same conclusion for the heart-muscle. During an investigation in

1917 (7) I was able to confirm the experiments of BIEDERMANN and ENGELMANN. I rejected however their conclusion. I demonstrated indeed that the swollen part of the muscle or the auricles had already shortened on account of the edematous shortening already present and that therefore a stimulus did not have any effect. I proved this by means of experiments. In water rigor — owing to the swelling of the muscle-fibres — a contraction of the ventricle could not be recorded on account of edematous swelling. The electrograms of the ventricle however continued from which we may conclude that the processes of the systoles continued rhythmically. I performed similar experiments with the gastrocnemius muscle. From this investigation it was also evident that the mechanical and electrical responses were inseparable. The literature of this subject is mentioned in a communication by FULTON (8), who came to the same conclusion in an interesting investigation.

I proceeded from my above mentioned investigation and I arrived at the conclusion that Dr. QUERIDO did therefore not see any mechanical response after an induction shock during the contracture, *because at that moment the muscle was already shortened.*

This conclusion was quite right. I proved it in the following way:

In former investigations (9, 10, 11, 12 and 13) I showed that the M. gastrocnemius of the frog consists of two groups of muscle fibres. The one group is innervated by the 8<sup>th</sup>, the other by the 9<sup>th</sup> spinal nerve. Both nerves were tied and cut near their exit from the vertebrae. Both nerves were placed on separate electrodes, which were connected by means of a commutator to the secondary circuit of an induction apparatus. I was thus able to stimulate each of the nerves separately by means of an induction shock. The tendon of the gastrocnemius was connected with a recording lever for the registration of the contraction curves. Then three to five drops of acetate of veratrin 1 p. c. were injected into the dorsal lymph sac. As soon as the poisoning process had proceeded sufficiently, the experiment began. At first both roots were stimulated separately and when both curves arrived about at the same height, the experiment was continued. After a pause of five minutes, the 8<sup>th</sup> spinal nerve was stimulated by an induction shock. A distinct veratrin curve followed. During the contracture both nerves were stimulated alternately. The result is shown in Fig. 2. We see that at the maximum height of the contracture an induction shock of the 8<sup>th</sup> as well as of the 9<sup>th</sup> spinal nerve does not give any response. At a later period, when the height of the curve has decreased, an induction shock of either of both nerves does give response. I obtained the same effect if the ninth spinal nerve was stimulated first as is shown in the second part of Fig. 2. From this experiment it is evident, that during the veratrin contracture, produced after an induction shock of the 8<sup>th</sup> spinal nerve, an induction shock of the 9<sup>th</sup> does not give any mechanical response, because the muscle is then already shortened. It besides the already contracting fibres more

begin to contract, the height of the mechanical curve does not increase. This would certainly happen if the two groups of muscle fibres were arranged one above the other.

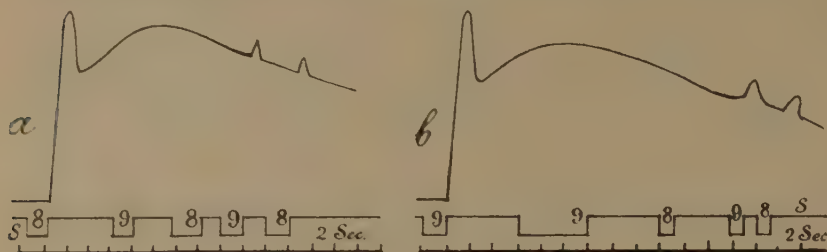


Fig. 2.

*a.* At 8 a veratrin contraction of the *M. gastrocnemius* is caused by an induction shock of the 8th spinal nerve. During the contracture the 8th and the 9th spinal nerve are stimulated alternately. At the maximum height of the contraction neither after an induction shock of the 8th nor after an induction shock of the 9th spinal nerve does any response in the mechanogram appear. But as soon as the height of the contracture decreases it occurs.

*b.* The experiment of *a* is repeated here with the same preparation; the result is the same. Now, however, the veratrin curve is registered after an induction shock of the 9th spinal nerve. An upward deflection of the signal arises when the primary circuit is broken. Only the break shocks are sent in. This also applies to fig. 3 and 4.

If however one of the spinal nerves innervates only a small part of the muscle fibres of the *M. gastrocnemius* and the other spinal nerve a large part, a stimulus of the first nerve gives a small contraction curve and the same stimulus of the second one yields a big contraction curve if the same weight is raised in both cases. If I repeat the experiment with a similar preparation, I easily obtain a confirmation of my experiment if I first register the big curve. If however I first register the small one e.g. after an induction shock of the 8<sup>th</sup> spinal nerve, then during the maximum height of the contracture an induction shock of the 8<sup>th</sup> spinal nerve has no effect. An induction shock of the 9<sup>th</sup> spinal nerve however yields a large veratrin curve, as is clearly shown in Fig 3.

Now one might suppose that during a veratrin contracture produced after an induction shock of the 8<sup>th</sup> spinal nerve not only the group of muscle fibres, innervated by this nerve would be refractory but also the remaining part innervated by the 9<sup>th</sup> spinal nerve. This possibility however is excluded, because, as I have proved, both groups of muscle fibres contract insulated chemically and electrically after poisoning with veratrin (9).

Both groups of muscles-fibres therefore work independently of each other in a chemical as well as in an electrical sense. If either spinal nerve is stimulated rhythmically until the contractions are those of great fatigue and the other nerve is then stimulated, the contraction caused by it shows no sign of fatigue. Further recovery from fatigue which has been produced by one nerve is unaffected by fatigue stimulation of the other nerve. It was



therefore evident from my former investigations that during activity of one group of muscle fibres the other group cannot be refractory.

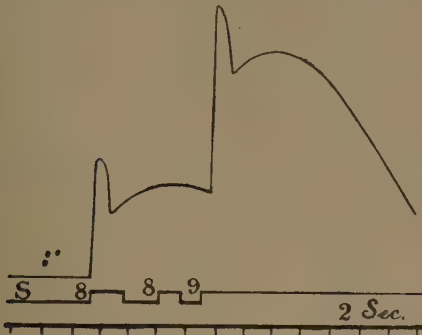


Fig. 3.

The same experiment as in fig. 2, but now the veratrin curve is small after an induction shock of the 8th spinal nerve. An induction shock of the 8th spinal nerve at the maximum height of the contracture does not have any effect now, an induction shock of the 9th spinal nerve however has effect because such a large veratrin curve is produced by it.

*It is thus evident, that by means of registration of the mechanical curves, as Dr. QUERIDO did, it is impossible to decide, whether during the contracture the muscle is refractory or not. Indeed an eventual shortening of the muscle cannot be registered mechanically during the contracture.*

I have therefore performed experiments, in which the mechanograms are registered as well as the electrograms. From these experiments it was clearly shown that during a maximum contracture induction shocks do not give any effect in the mechanogram but at the same time activity of the muscle does give a distinct deflection of the electrogram. As soon as the height of the contracture decreases, the mechanogram also shows a distinct shortening of the muscle after an induction shock. This effect is clearly shown in Fig. 4<sup>1</sup>). If however the contracture is

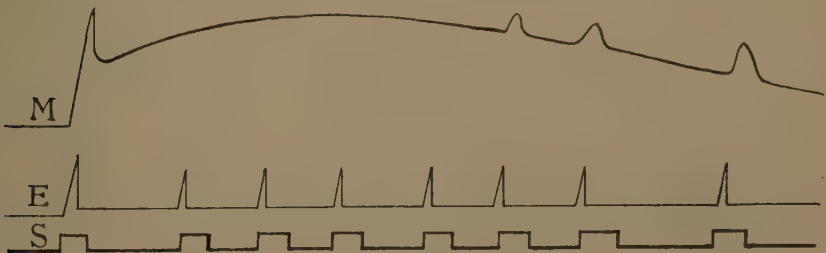


Fig. 4.

The sciatic plexus is placed on the electrode. At the top the mechanogram (M) is registered, at the bottom the electrogram curve (E). (One electrode is placed a little below the nervous aegator, the other on the tendon, shifting of the electrodes is prevented by fixing them to the muscle). At the maximum height of the contracture the induction shocks have no effect on the mechanogram, they do have effect on the electrogram curve.

<sup>1</sup>) For this investigation I used a platina string. For the purpose of this investigation it was sufficient.

smaller, then after each stimulus the response is also produced in the mechanogram. We conclude therefore from these experiments that during the veratrin contracture the skeletal muscles are not refractory. Dr. QUERIDO's conclusion is therefore wrong <sup>1)</sup>).

I shall mention more particulars of this investigation in a more elaborate publication and I shall then demonstrate that already without registration of the electrograms I was able to prove that during the whole contracture the muscle is not refractory. I should like to call your attention to one particularity, viz. that in the mechanogram a shortening is produced after an induction shock not only at the descending part but also in the ascending part of the contracture at the same time when at the maximum height of the contracture a stimulus does not give any affect.

### *Theoretical discussion.*

After the before mentioned investigations it is evident that the skeletal muscles are not refractory during the veratrin contracture. If an induction shock did not give any effect in the mechanogram, this was caused by the fact that at that moment the muscle was already shortened. Here we have to do with a similar experiment to that which was explained by me in 1917, with the skeletal muscles which had absorbed water. The conclusion of Dr. QUERIDO: "I consider, however, that the facts, which I mentioned about the refractory period of the muscle poisoned with veratrin, justify the conclusion that both parts of the curve are caused by the activity of one substratum and that consequently the theory of BOTTAZZI is not tenable", is therefore anticipated. If now, after my investigation, we should continue with Dr. QUERIDO's conception, we necessarily arrive at the opposite conclusion, viz. that the veratrin contracture is caused by the activity of another substratum than that in which the twitches take place. At the same time after my investigation all the conclusions drawn by Dr. QUERIDO about the absolute refractory stage, the relative refractory stage and about the determination of the size of the maximum stimulus by means of the refractory period are excluded.

Dr. QUERIDO draws a second argument in favour of his conception from the electrograms, which accompany the veratrin contracture. P. HOFFMANN (4) was the first who investigated the veratrin contracture by means of the string-galvanometer. He found during the contracture oscillations of the electrogram curve, which were most pronounced in the beginning of the contracture. These oscillations were much smaller after strong poisoning and then generally limited to the beginning of the contracture. The electrogram-curve afterwards became quite smooth, although the mechanical curve was still at the maximum. The oscillations,

---

<sup>1)</sup> The 24 curves which Dr. QUERIDO publishes on page 58 and 59 of his dissertation are therefore wrongly explained.

which P. HOFFMANN found after strong poisoning, were very small and became still smaller or disappeared when the stimulus was repeated several times.

In 1913 (2) my publication appeared on this subject. I also investigated the veratrin contracture by means of the string-galvanometer and *I only used a strong poisoning*. I used the monophasic leading off on purpose. To prevent shifting of the electrodes I registered the isometric curves besides the electrograms. I only made use of a silvered quartz string, which was very sensitive and which easily produced the rapid oscillations <sup>1)</sup>. Just as HOFFMANN I also found the rapid oscillations in the beginning of the contracture. These oscillations of my curves (2) were more pronounced than those of HOFFMANN's experiments in which he used a strong poisoning. These oscillations decreased or disappeared, when the poisoning was very strong or if the stimulus were repeated several times. Up to this point the results of HOFFMANN and myself agree. But I added one more to the before mentioned phenomena. I also found in my monophasic leading off a slow deflection of the string during the contracture. If we use the biphasic leading off, this slow deflection does not appear. I will explain this by a schema (in which I neglect the rapid oscillations, vide fig. 5). When we make use of the biphasic leading off, the muscle at first becomes electro-negative under the upper electrode and this electro-negativity continues as long as the contracture lasts. This causes the deflection A—B—C—D. A short time afterwards the muscle tissue under the other electrode becomes electronegative and this causes the deflection E—F—G—H in the opposite direction. The result is then a rapid biphasic deflection during the initial contraction and during the contracture the string remains in the zero in consequence of addition of both slow deflections. In the monophasic leading off therefore after the initial rapid deflection A—B the slow one C—D occurs because from this deflection the deflection E—F—G—H is not subtracted. (Vide fig. 5.) The

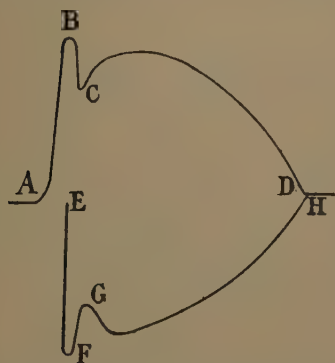


Fig. 5.

Schema of biphasical leading off in a muscle poisoned with veratrin. The electronegativity at the upper electrode is represented by the curve A—B—C—D, that at the lower one by the curve E—F—G—H. The result is a rapid biphasic curve in the beginning. Later on the string remains in the zero under the influence of two negativities with opposite sign.

<sup>1)</sup> Vide page 145 of my publication.

difference between the results of HOFFMANN and myself is only caused by the different leading off and not as Dr. QUERIDO, who also used the biphasic leading off suggests by the fact that the string which I used, was not sensitive enough <sup>1)</sup>. In his experiments Dr. QUERIDO like HOFFMANN and myself obtained oscillations which also were more distinct in the beginning of the contracture. The slow deflection was wanting of course, in any case it was not pronounced and always in the same direction. In some of his curves the string deflects a little in one direction, in other curves a little to the opposite direction. These different deviations are probably caused by impurities of the experiment.

Dr. QUERIDO rather remarkably does *not* mention my investigations of 1913 and he has forgotten to mention the rapid oscillations which I described in many of my curves <sup>2)</sup>. Moreover I mention from the text of my previous publication (1913): "Die zweite langsame Schwankung bot gewöhnlich in ihrer ersten Strecke sekundäre Schwankungen, die bisweilen sehr ausgesprochen waren." I already mentioned in short, that the oscillations which HOFFMANN considered as the main point, were considered by me as accidental and I am still of this opinion. Muscle contractures indeed are often accompanied by fibrillar contractions and we also see them after poisoning with veratrin especially after weak poisoning. In my opinion these oscillations are caused by the fibrillar contractions. The magnified shadow of the muscle, which Dr. QUERIDO photographed obtains the small oscillations by the same cause. The fibrillar contractions are only more pronounced in weak poisoning with veratrin and less pronounced in strong poisoning. At the same time it is easily explainable that in strong poisoning the oscillations are less pronounced than in weak poisoning <sup>3)</sup>. Dr. QUERIDO's second argument viz. the presence of oscillations is insufficient to justify his conception (tetanic contraction). This argument was not new and had already been used by HOFFMANN in his investigation, which I knew in 1913 and which I mentioned circumstantially.

#### LITERATURE.

1. QUERIDO A. Over 'de werking van veratrine op dwarsgestreept spierweefsel. Amsterdam. Em. Querido's Uitgevers-Maatschappij. 1926.
2. DE BOER S. Ueber das Elektrogram der veratrinisierten Muskeln. Zeitschr. f. Biologie Bd. 61. 1913. p. 143.

<sup>1)</sup> As I already mentioned, I used a sensitive silvered quartzstring, which could move very rapidly. The tension as described in my annotations was usually such that 1mV caused a deflection of 1 to 2mM. In some experiments 1 mV caused a deflection of 3 or 4 mM. I have already mentioned in my former communication (1913) that with the slack strings the slow deflection especially is more pronounced than the rapid one.

<sup>2)</sup> I emphasize here that HOFFMANN as well as I saw that the oscillations disappeared under certain circumstances.

<sup>3)</sup> The oscillations were still very distinct in the strong poisoning (4 to 10 drops of acetate of veratrin 10%) which I used, but from the investigation of HOFFMANN we know that in very weak poisoning the oscillations are much more pronounced than in strong poisoning.

3. BOTTAZZI PH. Ueber die Wirkung des Veratrins und anderer Stoffe auf die quer-gestreifte atriale und glatte Muskulatur. Archiv f. Physiol. 1901. p. 377.
  4. HOFFMANN P. Ueber die Aktionströme des mit Veratrin vergifteten Muskels. Zeitschr. f. Biologie Bd. 58. 1912. p. 55.
  5. BIEDERMANN W. Beiträge zur allgemeinen Nerven- und Muskelphysiologie. Sitzungsber. der Math. Naturwissensch. Classe der Akademie der Wissensch. in Wien. Bd. 97. 3. Abt. 1888. p. 84.
  6. ENGELMANN TH. W. Beobachtungen und Versuche am suspendierten Herzen. II. Ueber die Leitung der Bewegungsreize im Herzen. Pflüger's Archiv Bd. 56. 1894. S. 149.
  7. DE BOER S. Are contractility and conductivity two separate properties of the skeleton-muscles and the heart? Verslag Kon. Akad. v. Wet. Versl. Deel 26 1917 p. 535 and Proc. Vol. 20 p. 554.
  8. FULTON J. F. The inseparability of mechanical and electrical Responses in skeletal Muscle. The americ. Journ. of Physiol. Vol. 75. p. 261. 1925.
  9. DE BOER S. The double innervation of the M. gastrocnemius. Journ. of Physiol. Vol. 60. 1925. p. 215.
  10. DE BOER S. The monosegmental innervation of the muscle fibers of the M. gastrocnemius of the frog. The Americ. Journ. of Physiol. Vol. 75. p. 285. 1925.
  11. DE BOER S. On the segmentation of skeletal muscles in the frog (*rana esculenta*) Royal Acad. Amsterdam, Proc. Vol. 28. p. 821. 1925 and Ueber die Verteilung der Muskel-segmente im M. gastrocnemius des Frosches. Pflüger's Archiv Bd. 211. p. 636. 1926.
  12. DE BOER S. Die segmentelle Innervation einiger Froschmuskeln. Zeitschr. für Biologie. Vol. 85, p. 471. 1927.
  13. DE BOER S. The Americ Journ. of Physiol. 1927.
-





[That nevertheless with solid-vapour or liquid-vapour  $\left(\frac{dp}{dt}\right)_{\text{coex.}}$  approaches to 0 at  $T=0$ , has also nothing whatever to do with the theorem of heat, but is only owing to this that in  $\left(\frac{dp}{dt}\right)_{\text{coex.}} = \frac{L}{T\bar{v}_3}$  the vapour volume  $v_3$  ( $v_1$  or  $v_2$  may be neglected by the side of  $v_3$  at low temperatures) then approaches  $\infty$  more strongly than  $T$  approaches 0. For with  $p\bar{v}_3 = RT$  follows  $\frac{1}{p} \left(\frac{dp}{dt}\right)_{\text{coex.}} = \frac{L}{RT^2}$ , hence (supposing  $L$  at these low temperatures independent of  $T$ )  $\log p = -\frac{L}{RT} + C$ ,  $p = C'e^{-L/RT}$ , so that  $v_3$  becomes  $= RT : p = C''Te^{L/RT}$ , in which  $L$  (the heat of sublimation or evaporation) remains finite throughout].

And the same thing applies to the melting heat  $Q$ , which can become  $=0$ , but not only at  $T=0$  (cf. § 3). For this quantity is not only determined by the difference of the *thermic* Energies, as NERNST and others believe, but also by the much more important difference of the *potential* Energies. The latter is often overlooked.<sup>1)</sup>

## II. The shape of the melting-point line in the neighbourhood of $T=0$ .

The difference of the *potential energies* just mentioned is never  $=0$ , unless the liquid and the solid state should be *identical*, but then there are of course not two coexisting phases. The truth on this point is, that it only appears from the experimental data that at about  $1^\circ$  abs.  $p$  passes through a *minimum*, where  $Q=0$ . But at  $T=0$   $Q$  will become  $=\alpha T \log T$  (as we shall see in § 3), so that (we henceforth omit the indication coex. with  $\frac{dp}{dt}$ )

$$\left(\frac{dp}{dt}\right)_{T=0} = \frac{Q_0}{T(v_2 - v_1)} = -\infty$$

(see Fig. 1 A), since at  $T=0$   $v_2 - v_1$  (at finite pressure) either remains finite, or (accidentally) might also be  $=0$ .

For all other substances the melting-point line will intersect the vapour line, so that the ordinary course of Fig. 1<sup>a</sup> will be obtained, with a triple-point at  $D$ , but in Helium no intersection takes place (in consequence of the circumstances set forth in my preceding Paper).

[At very high pressures  $v_2$  will more and more approach to  $v_1$ , hence  $\frac{dp}{dt} = \frac{Q}{T(v_2 - v_1)}$  will approach to  $\infty$ , because at ordinary and higher tem-

<sup>1)</sup> See with regard to the use of these shortened formulae by NERNST and others some examples in my Paper in These Proc. 27, p. 897—913, especially § 7: A *thermodynamic sophism* (p. 911—913).

peratures  $Q$  is finite positive, which consequently — if  $T$  remains finite — gives an asymptotic course to the vertical line  $T_{lim.}$ , in which  $T_{lim.}$  can be greater or smaller than the critical temperature].

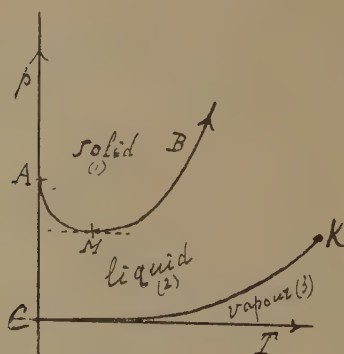


Fig. 1. (Helium)

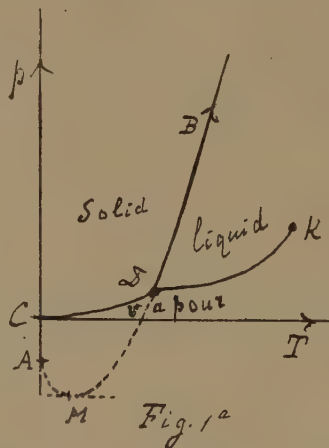


Fig. 1a

We will point out here, that in the case  $v_2 < v_1$ , which sometimes occurs (the liquid density greater than that of the solid substance), the course of the melting-point line will be just the reverse, f.i. as drawn in Fig. 2. Then  $\left(\frac{dp}{dT}\right)_{T=0}$  is  $= +\infty$ , and there will occur a maximum at  $M$ . It is self-evident, that in this case the melting-point line always intersects the vapour line.

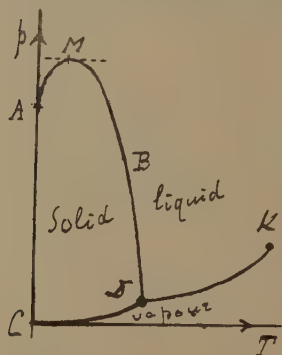


Fig. 2

The shape of the melting-point line in this latter case was fully described and derived by me in a series of 7 papers already more than 17 years ago<sup>1)</sup>.

### III. The course of the melting heat $Q$ .

At lower temperatures the following equation can evidently be written for this (the index 2 refers to the liquid), when  $E_0$  is the zero-point energy of the solid Helium:

$$Q = E_2 - E_1 = (k_2 T - E_0 - \theta_1 T^4) + (A_2 - A_1) + (P_2 - P_1) + p(v_2 - v_1), \quad (3)$$

<sup>1)</sup> „On the Solid State”. Cf. III, and IV, These Proc. of June 1909, p. 133 (Fig. 6 of the Plate). Also Arch. Néerl. (2) 15, p. 1–56 (1910) (Plate III, Fig. 6); also Chemisch Weekbl. 1910, N<sup>o</sup>. 50, p. 1–28 (Fig. 3) and 1912, N<sup>o</sup>. 37, p. 744–755 (Fig. 2). With regard to the ordinary case  $v_2 > v_1$ , I at first assumed critical endpoints on the strength of the now abandoned association theory, which seems now rather improbable to me.

in which the difference of the potential energies of the attractive forces, viz.  $A_2 - A_1$ , is evidently  $= \frac{a_1}{v_1} - \frac{a_2}{v_2}$ . Further the difference  $P_2 - P_1$  of the potential energies of the static repulsive forces will be  $= 0 - P_1$ , since this quantity does not exist in the liquid state. For there the molecules are not confined to definite positions of equilibrium. For

$P_1 = \int_v^{\infty} \frac{\lambda/v}{v-b} dv$  we find<sup>1)</sup> a somewhat complicated expression, which

of course disappears at  $v = \infty$ , and attains its greatest value, viz.  $+\infty$ , at  $v = v_{00}$  ( $p = \infty$ ). Also  $k_2$ , the specific heat at (infinitely great) constant volume of the liquid helium, remains finite in the neighbourhood of  $T = 0$ , but  $\sim$  in consequence of the decrease of  $a$  at decrease of the temperature<sup>2)</sup>  $\sim$  it is smaller than 3 at higher temperatures. For we have seen in our preceding paper on Helium, that in the case of liquid Helium no degradation of the Energy appears down to the lowest temperatures  $\sim$  in opposition to what many investigators suppose  $\sim$  so that the  $RT$ -law remains valid there. All the abnormal phenomena found for Helium can be accounted for by the dependence on the temperature of  $a$  and  $b$ .

If we now neglect  $\theta_1 T^4$  by the side of  $k_2 T$ , we get:

$$Q = k_2 T + \left( \frac{a_1}{v_1} - \frac{a_2}{v_2} \right) - (P_1 + E_0) + p(v_2 - v_1),$$

for which we may write at very low temperatures:

$$Q = k_0 T - C + p(v_2 - v_1), \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

since  $\left( \frac{a_1}{v_1} - \frac{a_2}{v_2} \right) - (P_1 + E_0)$  will vary very little with the temperature<sup>3)</sup>,

as the volumes remain almost constant;  $k_0$  having been written for the limiting value of the specific heat at low temperatures.

When at the melting equilibrium the thermodynamic potentials of the two phases are put equal, it follows from  $E_1 - TS_1 = E_2 - TS_2$ , that  $Q = E_2 - E_1$  must also be  $= T(S_2 - S_1)$ .

Now  $S_2 = k_2 \log T + R \log(v_2 - b_2) + (S_2)_0$ , while  $S_1 = \frac{4}{3} AT^3$ . If  $v_2 - b_2 = RT : (p + a/v_2^2) = RT : a/v_2^2$  is written, we get the following equation:

$$Q = T \left[ k_2 + R \log T + R \log R - R \log a/v_2^2 + (S_2)_0 - \frac{4}{3} AT^3 \right],$$

<sup>1)</sup> Cf. These Proc. 29, N<sup>o</sup>. 4, p. 502 (1926), Footnote 1. The quantity  $\lambda$  is the same as that which occurs in the equation of state (2).

<sup>2)</sup> These Proc. 29 N<sup>o</sup>. 10, p. 1303-1334.

<sup>3)</sup> As  $a$  is a function of the temperature, and according to our preceding paper  $= a_0 + \alpha T - \gamma T^2$ , in the expression for the Energy  $\frac{a}{v}$  must be replaced by  $\frac{a - Ta'}{v} = \frac{a_0 + \gamma T^2}{v}$ , which will differ little from  $\frac{a_0}{v_0}$  in the neighbourhood of  $T = 0$  ( $a' = \left( \frac{da}{dt} \right)_v$ ).

for which at very low temperatures may be written, neglecting the term with  $T^3$ :

$$Q = T[(k_0 + R) \log T + C], \quad . \quad . \quad . \quad . \quad . \quad (3^b)$$

when  $k_0$  is the limiting value of the specific heat of the liquid Helium at  $T=0$ .

From this it is clearly seen, that at  $T=0$   $Q$  indeed approaches to  $T(k_0+R) \log T$ , i.e. to  $0 \times -\infty = 0$ , but  $\frac{Q}{T}$  to  $(k_0+R) \log T = -\infty$ , so that also  $\frac{dp}{dt}$  will approach  $-\infty$  (see Fig. 1), and *not* to 0, as KEESOM assumes. At about  $1^\circ$  abs.  $Q$  is *again*  $=0$ , because then  $(k_0+R) \log T$  is  $= -C$ ; but now also  $\frac{Q}{T}$  is  $=0$ , hence  $\frac{dp}{dt} = 0$ , which has been found experimentally by KEESOM. At still higher values of  $T$  both  $Q$  and  $\frac{dp}{dt}$  become positive.

The same thing follows, of course, also from (3<sup>a</sup>), when the course of  $p$  and of  $Q$  according to (3<sup>b</sup>) is taken into consideration. But it is better to use the equation (3<sup>a</sup>) for the calculation of the vapour pressure at low temperatures. From

$$p = \frac{Q + C - k_0 T}{v_2 - v_1}$$

follows at  $T=0$ , when  $Q=0$ , that  $p_0 = C : (v_2 - v_1)$ . Inversely the quantity  $C = \left( \frac{a_1}{v_1} - \frac{a_2}{v_2} \right) - (P_1 + E_0)$  at  $T=0$  might be calculated from  $p_0$  and  $v_2 - v_1$ .

If, as is often done, the *potential* energies has been neglected,  $C$  would have become  $= -E_0$ , and in contradiction to observation, the limiting pressure  $p_0$  would have become negative at  $T=0$ !

We see accordingly that here the theorem of heat (or rather the  $T^4$ -law of DEBIJE) does not play any essential part, and that the fact, that  $\frac{dp}{dt}$  becomes 0 at  $\pm 1^\circ$  abs., by no means involves that  $\frac{dp}{dt}$  would also have to disappear at  $T=0$ , which — we already pointed this out in § 1 — is not required by any theorem. If Prof. KEESOM should succeed in descending to still lower temperatures, he will very probably be able to observe after the minimum at  $1^\circ.2$  abs. a change of the sign of  $\frac{dp}{dt}$  in the melting point line (from  $M$  to  $A$ ). This would at the same time be an interesting confirmation of my theory.<sup>1)</sup>

*Tavel sur Clarens, Suisse, Dec. 2, 1926.*

<sup>1)</sup> Cf. a later Article in a following number of these Proc. 1927, where the whole problem is treated in a more ample way.



**Chemistry.** — *Roentgenspectrographic Investigations on the Structure of the Artificial Ultramarines and the Problem concerning their Relations to the Minerals Hauyne, Nosean, Sodalite, Lazurite and Nephelite.* By Prof. Dr. F. M. JAEGER, Dr. H. G. K. WESTENBRINK and Mr. F. A. VAN MELLE.

(Communicated at the meeting of January 29, 1927).

§ 1. Notwithstanding the extensive technical experience gradually obtained in the preparation of the *ultramarine*-dyes during almost a hundred years, and notwithstanding the very great number of investigations made with the purpose to elucidate the true structure of these remarkable compounds, — it must be acknowledged, that the problem concerning the true chemical nature of the *ultramarines* still remains unsolved.

After CLEMENT and DESORMES <sup>1)</sup> in 1806 gave the data of a complete analysis of the blue ultramarine, GUIMET <sup>2)</sup> and GMELIN <sup>3)</sup> in 1828 discovered the methods allowing the artificial preparation in truly unlimited quantities of the precious dye formerly only obtained from *lapis lazuli*. In this case also chemical synthesis has finally surpassed nature, because experience has taught, that ultramarines of all kinds of colour can be obtained <sup>4)</sup> by mere variations of the applied method, by which the colouring matter was prepared by heating with *sodium-sulphide* or with a mixture of *sodium-sulphate* and coal some *sodium-aluminium-silicates* of a composition analogous to that of *nephelite*. Now, besides the original blue ultramarine, also *yellow*, *green*, *pink*, *red* and *violet* ultramarines are prepared; and within each of these colour-limits, a great number of different modifications may, moreover, be obtained, which differ from each other only by slight gradations of colour or by their particularly warm glow.

The occurrence of the blue ultramarine-dye in *lazurite* and of the blue colour in many *hauynes*, *noseans* and in some *sodalites*, have since long drawn the attention of mineralogists to the fact, that presumably a closer constitutional relation may exist between the natural *ultramarine* on the one side, and the remarkable minerals mentioned on the other, which minerals hitherto were considered to belong to one and the same family. More particularly BRÖGGER and BÄCKSTRÖM <sup>5)</sup> emphasized

<sup>1)</sup> J. K. CLÉMENT and DESORMES, Ann. de Chim. et Phys. 57, 317, (1806).

<sup>2)</sup> J. B. GUIMET, Bull. de la Soc. d'Encouragement, etc., (1828).

<sup>3)</sup> G. C. GMELIN, Abh. Naturw. Ges. Württemberg, (1828), 2, 191; SCHWEIGER'S Polyt. Journ. 54, 360.

<sup>4)</sup> See also R. HOFFMANN: „Ultramarin“, Braunschweig, (1902); L. BOCK, „Die Konstitution der Ultramarine“, Braunschweig, (1924); idem, „Die Fabrikation der Ultramarinfarben“, Halle, (1918); F. SINGER, „Die künstlichen Zeolithe“, Diss. Berlin, (1911). In these works and also in C. DOELTER's „Handbuch der Mineral-Chemie“, Band 2, 2, p. 295, 286 and 229—261, (1917) a detailed review is published of the literature concerning these compounds.

<sup>5)</sup> W. C. BRÖGGER and K. BÄCKSTRÖM, Zeitschr. f. Kryst. und Miner. 18, 209, (1890).

the mutual relations of these compounds as a result of their very detailed investigations on the *alkali-* and *calcium-garnets*. It was for this reason, that during our ROENTGENspectrographic work on the artificial ultramarines, we simultaneously started the examination of the said minerals after the same experimental method. The results of all these investigations are published in the present paper.

According to BRÖGGER and BÄCKSTRÖM, *sodalite*, *hauyne* and *nosean* must all be placed in one and the same group of minerals, namely that of the *alkali-* and *alkali-calcium-garnets*. According to these authors, *sodalite* would surely not possess hexakisoctahedral symmetry: as well the corrosion-figures obtained by them, as also other particulars of the rhombicdodecahedrally shaped crystals, seem to indicate a hexacistetrahedral symmetry and a twin-formation parallel to  $\{100\}$ . They attribute the same symmetry, moreover, to *nosean* and *hauyne*, — which also crystallize in most cases in the form of rhombicdodecahedra, — by drawing attention to the great analogy which exists between these minerals and *sodalite*, and to the octahedral forms with furrowed edges, as observed by VOM RATH. This last argument, however, does not hold <sup>1)</sup>, as may be seen from the analogous phenomenon in the case of *diamond*. In the case of a colourless *nosean* we observed, moreover, that the crystal accidentally broke off exactly parallel to  $\{111\}$ , while *sodalite* shows a cleavage parallel to  $\{110\}$ ; it is, however, yet hazardous to assume a cleavability parallel to  $\{111\}$  in *nosean*. In the following page it will become clear, that *sodalite* has, indeed, a structure, which is quite different from that of *nosean* and *hauyne*; so that it can by no means be placed in the same group with the others, although BRÖGGER and BÄCKSTRÖM believed to have proved, that *lazurite* is a mixture of *sodalite*, *hauyne* and natural *ultramarine*.

As regards the artificial *ultramarines*, — microscopical examination has proved, that they are optically isotropous, like the natural product, and that they crystallize in the shape of microscopical, very minute rhombicdodecahedra <sup>2)</sup>. Furthermore, the *sodium* of the ultramarines appears to be easily replaceable by other bases <sup>3)</sup>, e.g. by the oxides of the metals *K*, *Li*, *Ca*, *Ba*, *Mg*, *Zn*, *Ag*, etc., — as is the case also in the *permutites* and some *zeolites*. As is the case in the *zeolites*, the *sodium*-atoms in these compounds are evidently highly mobile; moreover, SINGER <sup>4)</sup>

<sup>1)</sup> Conf. F. M. JAEGER. „Inleiding tot de Studie der Kristalkunde. Groningen, (1924), p. 178, 179.

<sup>2)</sup> See also F. FISCHER, DINGLER's Polyt. Journal, 221, 562, (1876); T. MOREL, Mon. Scientifique, (1879), 785; C. GRÜNZWEIG and R. HOFFMANN, Ber. d. d. Chem. Ges., 9, (1876), 864; W. C. BRÖGGER and H. BÄCKSTRÖM, loco cit., p. 242.

<sup>3)</sup> K. HEUMANN, Ber. d. d. Chem. Ges. 10, (1877), 991; P. G. SILBER, ibid., 14, 941, (1881); DE FORCRAND and BALLIN, Bull. Soc. chim., 30, 112, (1878); Compt. rend., 88, 30, (1879); K. HEUMANN, Ann. d. Chem. u. Pharm., 201, 262, (1880); J. SZILASI, Ann. der Chemie und Pharm., 251, 97, (1889).

<sup>4)</sup> F. SINGER, „Ueber künstliche Zeolithe“, Diss. Berlin, (1911).

demonstrated, by means of direct synthesis, that there exists a close relation between *permutites* and *zeolites* on the one hand, and the *ultramarines* on the other hand. Also derivatives of the ultramarines are known, in which *aluminium* is substituted by *titanium*, *silicon* by *boron*, etc., and in which *selenium* and *tellurium* hold the place of the *sulphur*-atoms<sup>1)</sup>. On treating the ultramarines with acids, about one third of the *sulphur* present is made free as  $H_2S$ , while the remaining part is precipitated in the form of elementary *sulphur*.

COMPOSITION OF SOME STUDIED ULTRAMARINES.				
GUIMET I; Violet	GUIMET II; Red	GUIMET III; Blue	GUIMET IV; Green	Germ. Ultram. VIII; Blue
Specific weight: $d_{250} = 2.34$ Al = 14.25 % Si = 19.33 % S = 10.40 % Atomic ratio: Al:Si:S = 6:6.86:3.72	Specific weight: $d_{250} = 2.36$ Al = 15.53 % Si = 19.79 % S = 9.65 % Atomic ratio: Al:Si:S = 6:7.30:3.13	Specific weight: $d_{250} = 2.35$ Al = 13.10 % Si = 19.38 % S = — Atomic ratio: Al:Si = 6:8.52	Specific weight: $d_{250} = 2.27$ Al = 16.54 % Si = 17.21 % S = 7.98 % Atomic ratio: Al:Si:S = 6:6.06:2.44	Specific weight: $d_{250} = 2.37$ Al = 14.01 % Si = 18.33 % S = — Atomic ratio: Al:Si = 6:7.50

Some specific weights of German ultramarines were, in addition, also determined:

D. V.; dark violet:  $d_{250} = 2.41$ . D. VI.; violet:  $d_{250} = 2.37$ . D. VII.; pink:  $d_{250} = 2.48$ .

§ 2. For this investigation we had a number of differently coloured products at our disposal from the *Usines d'Outremer-GUIMET* in *Fleurieu* s/Saône, namely: GUIMET-green N°. 9210; blue N°. 7553; pink N°. 8219 and violet OVR; furthermore, a number of red, green and blue preparations from the *Vereinigte Ultramarin-Fabriken* in *Cologne*, which most kindly were supplied to us through the medium of Dr. C. O. LEVERKUS; they were partially prepared after the *sulphate*-, partially after the *soda*-, partially after the mixed *sulphate-soda*-method. With the exception of the green ultramarines, all the products investigated appeared to belong to the series of compounds with *high silica-content*; a number of analytical data obtained with the carefully dried materials, and a series of determinations

<sup>1)</sup> E. GUIMET, Bull. Soc. chim., 27, 480 (1877); Ann. d. Chim. et Phys. (5), 13, 102, (1878); D. MOREL, ibid. 28, 522, (1878); J. HOFFMANN, Koll. Zeits., 10, 275, (1911).

of the specific weights of the same preparations at 25° C., are recorded here. (See table).

From these data, which were all obtained in repeated and agreeing analyses with the aid of most carefully dried ultramarine-species, it becomes clear, that there cannot be spoken here of a really „constant” composition of these products: even the two blue preparations *G. III* and *D. VIII*, — which contain both a high percentage of silica, — appear to deviate appreciably from each other in this respect. Neither is there a simple stoichiometrical ratio of the *Al*-, *Si*-, and *S*-atoms present, it being, therefore, impossible to give here rational molecular formulae for these compounds. In literature the following data are given for the blue ultramarines of high silica-content: *Al* = 12,70 till 13,93 %; *Si* = 19,07 till 19,53 %; *S* = 13,44 till 14,09 %; while, for an ultramarine of the formula:  $Al_4 Na_6 Si_6 S_4 O_{20}$  these numbers should be: 12,53 %, 19,49 % and 14,85 % respectively. It is evident that the chemical composition of these dyes must be variable within rather wide limits.

For the ultramarines with a high silica-content, the empirical formulae seem to oscillate more or less round:  $Na_6 Al_4 Si_6 O_{20} S_4$  for the *blue* and  $Na_3 Al_4 Si_6 O_{22} S_3$  for *red* ultramarine; for *green*, which belongs to a series of „moderate” sulphur-content, the composition oscillates round:  $Na_9 Al_6 Si_6 O_{24} S_3$  or more correctly round:  $Na_8 Al_6 Si_6 O_{24} S_2$ . The sulphur-content is highest in the ultramarines prepared after the soda-method, lowest in those made by means of the sulphate-process.

With respect to the minerals studied here, it must be remarked, that they were taken from different localities: colourless *sodalite* from the *Monte Somma*, *Vesuvius*; blue *hayne* from the same spot, from the *Laacher See* in the *Eifel* and from *Niedermendig*; colourless *nosean* from the two last mentioned localities. Some of these crystals were kindly put at our disposal by Prof. RUTTEN of Utrecht and Prof. ESCHER of Leyden, to whom we also in this place wish to express once more our gratitude for their kind help. The ROENTGENSpectrographic study of these minerals, and more particularly the interpretation of their rotation-spectrograms, proved to offer extraordinary difficulties, as a consequence of the fact, that all these crystals were not only very small and often inhomogeneous, but, moreover, that as well in the case of *hayne*, as in that of *nosean* and *sodalite*, they were almost always twins. In connection with the great identity-distances of their structures in the direction of the axis of rotation, the last mentioned circumstance made a rigorous analysis of the diffraction-images obtained often very difficult and sometimes quite impossible. With the exception of the special case, that the revolution of the crystal occurred round an axis perpendicular or parallel to the twinning-plane, the analysis of the spectrograms obtained could even be considered as quite hopeless. If e.g. a twin parallel to (111) be rotated round an axis in a direction, which has the symbol, [111] for the first of the two apparent individuals of the twin, then no dist-

turbance of the spectrogram occurs, in so far as the determination of the identity-distance in that direction  $[111]$  by means of the successive hyperbolae is considered, because the said direction has the symbol  $[11\bar{1}]$  in the second apparent individual of the twin. But if the same twin be rotated round axes in directions as  $[110]$ ,  $[\bar{1}11]$ ,  $[\bar{1}\bar{1}1]$ , etc. in the first apparent individual, — these directions are resp.  $[114]$ ,  $[\bar{1}51]$ , and  $[115]$  in the second individual. As the identity-distances in a direction  $[h k l]$  in the cubic system are c. p. proportional to  $\sqrt{h^2 + k^2 + l^2}$ , — the succession of the hyperbolae in the spectrogram obtained will appear to be disturbed until hardly recognisable, if the crystal be rotated round axes in the directions mentioned above, and especially in the case of such minerals as are studied here, in which the mutual distances of the successive hyperbolae in the image are, after all, already very small.

§ 3. In the first place a series of powder-spectrograms of the *ultra-*

TABLE I.  
Powder-spectrogram of Nosean, Hauyne and eight different Ultramarines.

No. of line:	Intensity (estimated):	2 l in mm.:	$\lambda$ :	$\frac{\theta}{2}$ :	$\sin^2 \frac{\theta}{2}$ (observed):	$\sin^2 \frac{\theta}{2}$ (calculated):	Indices $\{h k l\}$
1	1	16.8	$\alpha$	$9^\circ 37'$	0.0279	0.0284	(200)
2	1	19.0	$\beta$	$10^\circ 53'$	0.0356	0.0352	(211)
3	8	21.0	$\alpha$	$12^\circ 2'$	0.0433	0.0426	(211)
4	5	27.0	$\alpha$	$15^\circ 28'$	0.0711	0.0710	(310)
5	5	29.7	$\alpha$	$17^\circ 4'$	0.0861	0.0852	(222)
6	1	34.5	$\alpha$	$19^\circ 49'$	0.1149	0.1136	(400)
7	3	36.5	$\alpha$	$20^\circ 58'$	0.1280	0.1278	(330)
8	3	44.5	$\alpha$	$25^\circ 33'$	0.1860	0.1846	(431)
9	2	49.5	$\alpha$	$28^\circ 25'$	0.2265	0.2272	(440)
10	1	51.3	$\alpha$	$29^\circ 27'$	0.2417	0.2414	(530)
11	1	53.0	$\alpha$	$30^\circ 22'$	0.2556	0.2556	(600)
12	1	54.8	$\alpha$	$31^\circ 24'$	0.2715	0.2698	(532)
13	1	59.5	$\alpha$	$34^\circ 9'$	0.3151	0.3124	(622)

Radius of camera: 25 mm.;  $V = \text{ca. } 55000$  Volt; time of exposure: 3 hours.

Wave-length:  $\lambda_{Cu} = 1.540$  A.U.  $\lambda_{Fe} = 1.388$  A.U.

Quadratic equation:  $\sin^2 \frac{\theta}{2} = 0.0071 (h^2 + k^2 + l^2)$ ; from which follows:

$a_0 = 9.13$  A.U.



*marines* GUIMET I—V and of a number of the ultramarines of German origin was prepared after HULL-DEBIJE's method, and, moreover, such of *nosean*, *hauyne* and *sodalite*. The remarkable result was obtained, that quite independent of the colour and the chemical composition of these ultramarines, all these spectrograms appeared to be completely identical, and, moreover, to be also quite the same as those for *nosean* and *hauyne*. On the other hand, *sodalite* gave a spectrogram, which, as well with respect to the situation of the diffraction-lines, as to their intensities, was quite different from them. Evidently no appreciable influence on the localisation and intensities of the diffraction-images is exerted here by the special circumstance, that, — as in *hauyne*, — the sodium-atoms of *nosean* are partially substituted (for about half their number) by calcium-atoms; or that the ultramarines investigated contain 3, 6, 8 or 9 atoms of sodium; or that the  $(\text{SO}_4)$ -group in *nosean*, in the transition to the ultramarines, is substituted by two, three or four sulphur-atoms. None of these changes appears to have any detectable influence whatsoever on the intensity or on the special situation of the diffraction-images.

A review of the complete analysis of these powder-spectrograms is given in the preceeding table; the values of  $\sin^2 \frac{\theta}{2}$  are calculated by means of the exact constant of the quadratic equation, as it was later-on accurately determined by means of BRAGG-spectrograms<sup>1)</sup>.

From these data it becomes evident, that in no case  $(h+k+l)$  is an odd number; the fundamental BRAVAIS' grating of the structure must, therefore, be the *body-centred* one. As will be shown, this conclusion is confirmed by all later experiments. As the specific weights of the ultramarines used are known ( $\approx$  about: 2.36), as well as those of *nosean* and *hauyne* (about: 2.40), — the number of "molecules" of these compounds per elementary cell can be calculated in the usual way.

For *nosean* of the simplest formula:  $\text{Na}_5 \text{Al}_3 \text{Si}_3 \text{O}_{16} \text{S}$ , this number appears to be: 2, and for the *ultramarines* of the formulae mentioned above, it is: 1; so that in every case the mass contained in the fundamental cell corresponds to six silicon-atoms in the different formulae.

§ 4. After the identity of the structures of *nosean*, *hauyne* and all *ultramarines* had been demonstrated in this way, we endeavoured to confirm the conclusions hitherto drawn with respect to the structure of these compounds, by means of rotation-spectrograms of *nosean* and *hauyne*, and to calculate accurately the dimensions of the grating by the aid of spectrograms prepared after BRAGG's method.

Originally *hauyne* gave in no case any useful rotation-spectrograms;

<sup>1)</sup> Still better spectrograms of ultramarine were afterwards obtained with a new and improved camera. No less than 32 different diffraction-images could be measured on these films. The results obtained, however, have fully confirmed the conclusions drawn in the above. In a following paper the new data will be discussed in detail.

but later-on very good spectrograms of this kind were obtained, which will be discussed in the next paper. The crystal used, which seemingly had the shape of a perfect rhombicdodecahedron, appeared to be a twin, which, if rotated round [111] or [110], gave no analysable rotation-spectrograms. However, a BRAGG-spectrogram on (110) could be obtained, *rocksalt* parallel to (100) being simultaneously used for the purpose of comparison. Observed value:  $\frac{\theta}{2} = 6^\circ 52'$ ; from which follows:  $d_{(110)} = 6.44$  A.U., and, therefore,  $d_{(100)} = 9.11$  A.U. That the "reflection" on (110) is really of the *first* order, was confirmed later-on experimentally.

The *nosean*-crystals were rhombicdodecahedra, which were strongly elongated parallel to one of their trigonal axes; they appeared to be twins parallel to (111). If such a crystal be rotated round its longest axis, a normal spectrogram may be expected beforehand, as was already emphasized in the above. The distance between the first hyperbola in the spectrogram to the centre of the photographic image was 8 mm.; as in this case the distance of the photographic plate to the crystal was: 40 mm. here, so  $\tan \mu = 0.20$  and, therefore,  $\mu = 11^\circ 20'$ . From this value the identity-distance in the direction of a trigonal axis is found to be:

$$I_{[111]} = \frac{\lambda}{\sin \mu} = 7.84 \text{ A.U.}$$

A BRAGG-spectrogram on a face of {110} gave  $\frac{\theta}{2} = 6^\circ 51'$ ; from which  $d_{(100)}$  is again calculated to be: 9.13 A.U.; the formerly observed diffraction-line must, indeed, be of the 1<sup>st</sup> order.

The diagonal of the cubic cell is calculated from  $d_{(100)} = 9.13$  A.U. to be: 15.80 A.U., which is nearly  $= 2 \times 7.84$  A.U. From this follows, that the spacings in the direction [111] are *halved*, — which proves that the cubic cell is really the *body-centred* one. If the image on (110) had been of the *second* order, then, in accordance with the data of the rotation-spectrogram obtained by turning the crystal round [111], the bodily diagonal of the cubic cell would have to be *quartered*. This case, however, is excluded in each of the three types of cubic gratings.

A complete analysis of the rotation-spectrogram of *nosean* on turning round [111] is given in the following table. The values of  $\sin^2 \frac{\theta}{2}$  are calculated here by means of the quadratic equation:  $\sin^2 \frac{\theta}{2} = 0.0071(h^2 + k^2 + l^2)$ , — the coefficient: 0.0071 being determined from the BRAGG-spectrogram  $\left(\frac{\theta}{2} = 6^\circ 51'\right)$  already mentioned.

As the fundamental grating is the body-centred one and as rotation took place round [111], the sum of the indices of the principal spectrum must be equal to *zero*; that of the indices of the 1<sup>st</sup> hyperbola equal to 2; that of those of the 2<sup>nd</sup> hyperbola equal to 4; that of the indices of

the 3<sup>d</sup> hyperbola equal to 6; etc. This now appears really to be the case; so once more the conclusions drawn previously are confirmed by this.

TABLE II.					
Analysis of the Rotation-spectrogram of Nosean, when rotated round [111].					
Distance between photographic plate and crystal = 40 mm.					
Distance between spot and centre of image in mm.:	$\frac{\theta}{2}$ :	$\sin^2 \frac{\theta}{2}$ (observed):	$\sin^2 \frac{\theta}{2}$ (calculated):	Indices $\{h\ k\ l\}$ :	Number of hyperbolae
18.0	12°7'	0.0441	0.0426	(2 $\bar{1}$ $\bar{1}$ )	Principal spectrum
30.0	18°26'	0.1000	0.0994	(3 $\bar{2}$ $\bar{1}$ )	
36.0	21°0'	0.1284	0.1278	(3 $\bar{3}$ 0)	
49.5	25°32'	0.1860	0.1846	(4 $\bar{2}$ $\bar{2}$ )	
62.5	28°0'	0.2304	0.2272	(4 4 0)	
18.0	12°7'	0.0441	0.0426	(2 1 $\bar{1}$ )	I
24.0	15°29'	0.0731	0.0710	(3 0 $\bar{1}$ )	
41.5	23°2'	0.1531	0.1562	(3 2 $\bar{3}$ )	
49.5	25°32'	0.1860	0.1846	(4 1 $\bar{3}$ )	
66.0	29°23'	0.2407	0.2414	(3 $\bar{3}$ $\bar{4}$ ) or (50 $\bar{3}$ )	
17.5	11°49'	0.0419	0.0426	(2 1 1)	II
24.0	15°29'	0.0713	0.0710	(3 1 0)	
33.0	19°46'	0.1144	0.1136	(4 0 0)	
30.0	18°26'	0.1000	0.0994	(3 2 1)	III
36.0	21°0'	0.1284	0.1278	(3 3 0)	
49.5	25°32'	0.1860	0.1846	(4 3 $\bar{1}$ )	
41.5	23°2'	0.1531	0.1562	(3 3 2)	IV
49.5	25°32'	0.1860	0.1846	(4 3 1)	
Wave-length. $\lambda_{Cu} = 1.540$ A.U.; $V = 55000$ Volt; time of exposure: 5 hours.					

Evidently all images  $\{hkl\}$  have the intensity = 0 in odd orders, when  $(h+k+l)$  is an odd number. This fact could be corroborated once more by means of a BRAGG-spectrogram on a cleavage-plane of

*nosean* parallel to (111): after a few minutes exposure the 2<sup>nd</sup> order of the diffraction-image was present already with a very considerable intensity, while even after exposure during a full hour, not the slightest trace of a 1<sup>st</sup> order-image could be detected.

§ 5. The next problem to be discussed, is that of the symmetry of *nosean* and *haugyne*, which, — as has already been said, — are commonly considered to possess hexacistetrahedral symmetry, an opinion also advocated by BRÖGGER and BÄCKSTRÖM. But there is no valid proof adduced for this, since the analogy of these minerals with *sodalite*, as brought to the fore by these authors as an argument, can no longer be maintained now. This will be seen from what follows, because we were able to prove, that the structure of *sodalite* is quite different from that of the two other minerals. As all arguments for the determination of the space-group of *nosean* and *haugyne* must be taken into account, it is, therefore, not allowed to make beforehand any restriction with respect to the special symmetry of these minerals, and it thus appears necessary to consider all crystal-classes of the cubic system as of the same importance.

It was proved, that the fundamental grating is the body-centred one, and that all spacings  $\{h\ k\ l\}$  are halved, if  $(h + k + l)$  is an odd number. If these facts be taken into account, then there are ten space-groups, whose special symmetry may eventually be attributed <sup>1)</sup> to both minerals considered here; namely

$$T^3; T^5; T_h^5; T_h^7; T_d^3; T_d^6; O^5; O^8; O_h^9; \text{ and } O_h^{10}.$$

in WYCKOFF's notation.

However, we can already immediately exclude from those the groups:  $T_h^7$ ,  $T_d^6$  and  $O_h^{10}$ , because on the BRAGG-spectrograms of  $\{110\}$  the first order image of  $\{110\}$  was observed with full certainty. Moreover, since also (200) was observed (see Table I) the group  $O^8$  too must be excluded.

*Nosean* and *haugyne* can, therefore, only have the symmetry of one of the following six groups:

$$T^3; T^5; T_h^5; T_d^3; O^5, \text{ or } O_h^9.$$

It must, furthermore, be remarked, that, — as the most probable formula for pure *nosean* is:  $\text{Na}_5\text{Al}_3\text{Si}_3\text{O}_{16}\text{S}$ , and as twice the number of atoms of this formula is present in the elementary cell, — the last one must contain a mass, the composition of which is in the simplest case expressed by:  $\text{Na}_{10}\text{Al}_6\text{Si}_6\text{O}_{32}\text{S}_2$ . In the analogously built *ultramarines* there is present in each case once the number of atoms which is expressed by the formulae for these compounds as given before; so that the elementary cell of all *ultramarines* also in each case contains six atoms of silicon.

Now, — what perhaps would appear to be the final choice between

<sup>1)</sup> W. T. ASTBURY and K. YARDLEY, Phil. Trans. Royal Soc. London, A, **224**, (1924) p. 254, 255, 256.

the six space-groups mentioned above, — in every case it can be proved on closer examination <sup>1)</sup>, that the possible arrangement of the equivalent twofold and sixfold places in the cell manifests itself in all six groups in *quite the same way*, — in  $T^5$  excepted, — in which group such places do not occur at all. For it can be shown, that the coordinates of these places in all those groups are:  $[000]$  and  $[\frac{1}{2}\frac{1}{2}\frac{1}{2}]$  for the *two-fold* places, while for the *sixfold* ones they are:  $[0\frac{1}{2}0]$ ,  $[00\frac{1}{2}]$ ,  $[\frac{1}{2}00]$ ,  $[\frac{1}{2}0\frac{1}{2}]$ ,  $[\frac{1}{2}\frac{1}{2}0]$  and  $[0\frac{1}{2}\frac{1}{2}]$  (See Fig. 1). All these coordinates are simply fixed

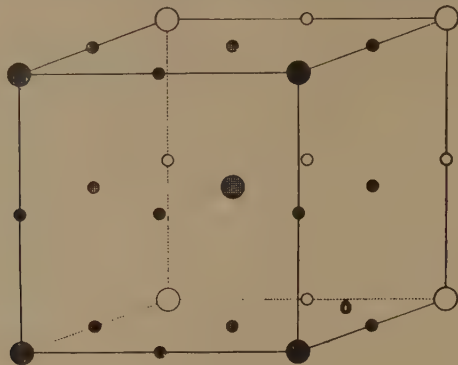


Fig. 1,

beforehand, because *they do not contain any variable parameter*. Moreover, in none of these groups can occur *univalent* or *fourfold* places; in all, however, there might be found *eight-*, *twelve-* and *twenty-four-fold* places, and in some of them also *sixteen-*, *forty-eight* and *ninety-six-fold* places. Now, because in the cells of *all* these compounds, *six silicon*-atoms appear to be present, there must certainly be present *sixfold* places in each case; so that we can also exclude beforehand the group  $T^5$  from the six mentioned above.

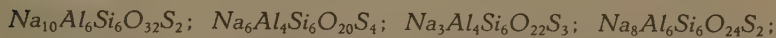
But the fact that the coordinates of the necessary *six-fold* places in the cell are completely fixed and that they do not contain any variable parameter, involves the impossibility of the simultaneous occurrence of other *sixfold* places in the structure, besides just that single complex of six points, the coordinates of which are mentioned above. If these six places are once occupied by six identical atoms of a certain kind, it is, therefore, excluded *a priori*, that six atoms of a second or third kind could simultaneously be present on six equivalent places within the same elementary cell.

Now, as *all* elementary cells evidently contain 6 Si-atoms, it is obvious to make the supposition, that it is these six *silicon*-atoms, which really

<sup>1)</sup> R. W. G. WYCKOFF, Publ. CARNEGIE-Institut. Washington, N<sup>o</sup>. 318, (1922), p. 122, 126, 129, 135, 148, 103.



occupy the only sixfold places available here, with the coordinates mentioned above<sup>1</sup>). If now, however, the composition of the masses present in the elementary cell of all these compounds be taken into consideration:



(Nosean)	(Ultramarine-blue)	(Ultramarine-red)	(Ultramarine-green)
	(rich in silica)	(rich in silica)	(poor in silica)

it will immediately be seen, that perhaps only the O-atoms possibly could be placed in the cell in some such way as:  $(12 + 8)$ ;  $(12 + 8 + 2)$ ; 24, (or  $16 + 8$ );  $(24 + 8)$  atoms; etc. It must be remarked, however, that the numbers of O-atoms indicated, are not very accurate, as all errors of the chemical analysis are accumulated in these numbers.

If 24 O-atoms must be placed as  $(16 + 8)$  atoms, the space-groups  $T_d^3$  and  $T_3$  would be excluded here also; and if the compounds studied here, were really holohedral, then the space-group  $O_h^9$  could most probably be attributed to them. But all this is mere speculation; any thing certain cannot be said about it.

However, — in what way the question may be considered or perhaps finally decided, — there is, after all, *one* fact, which is undeniably certain: namely, *it is quite impossible to attribute to an appreciable number of components of the total mass included within each elementary cell any definite place in the structure.* For these components: Na, Al, S, — and in the case of *hauyne* also for Ca, — no place whatsoever is any more available within the fundamental cell. While it is even quite uncertain, which atoms on the parameterless two- and sixfold places be really present, or which atoms are perhaps situated at the 8-, 12-, 16-, and 24-fold places, — it is, after all, fairly well possible to substitute the sixfold places by atoms of one and the same kind, e.g. by Si-atoms; but then every further possibility will simultaneously have disappeared for localising other complexes of six identical atoms, e.g. of  $Al_6$  or  $Na_6$ , because there are in reality *no other sixfold* places any more available within the cell, besides the six with the fixed coordinates, as mentioned in the above. Therefore, beyond all doubt there must be an appreciable number of atoms present within each fundamental cell, which can by no means be arranged in definite places of the periodical structure; they must in space be distributed at random, in a merely statistical way, and be scattered throughout the fixed, rigorously periodical structure of these crystals, which forms, as it were, their rigid skeleton.

In the same way GOLDSCHMIDT and THOMASSEN<sup>2</sup>) in the case of

<sup>1</sup>) It would e.g. be obvious to suppose that the two S-atoms, which in *nosean* and in the *ultramarines* are always present, be situated in the twofold places. However, our later experiments seem to indicate, that not these S-atoms, but rather two of the *sodium*-atoms must occupy these places.

<sup>2</sup>) V. M. GOLDSCHMIDT en L. THOMASSEN, Vidensk. Skrift., Christiania, (1923).

*cleveite* and *bröggerite* always found, by means of the ROENTGEN-ographic method, the periodical structure of  $UO_2$ , although the chemical composition of these minerals indicates, that they chiefly consist of  $U_3O_8$ , with as accessory components: *Pb*-, *Tho*-, *Yt*-, *Erb*-, *Ce*-, and *Fe*-derivatives, besides some  $H_2O$ . A certain amount of oxygen must, with the other accessory compounds, therefore, be statistical distributed within the  $UO_2$ -lattice, — exactly as occurs in our case of the ultramarines; so that the ROENTGEN-spectrograms do not reveal anything of the presence of these compounds, except the diffraction-lines, which correspond more particularly to the rigorously periodical structure of the  $UO_2$ -framework. An analogous explanation must doubtlessly be given for the facts observed by WYCKOFF, GREIG and BOWEN, that *sillimanite* and *mullite*, and also for these observed by WYCKOFF and MOREY, that a series of *sodium-calcium-metasilicates* of different chemical compositions, gave identical or hardly discernable ROENTGEN-spectrograms<sup>1)</sup>.

It is because of the special circumstances present in this case, — and more particularly because of the very favourable circumstance, that the coordinates of the *two*- and *sixfold* places of the structures considered here, are completely free from indefinite parameters  $u$ ,  $v$ , etc., — that we are inevitably compelled to the conclusion, that in the minerals studied here, — which formerly were considered as being “molecular compounds” between *silicates* and  $Na_2SO_4$ , — as well as in the *ultramarines*, — which were considered as analogous compounds between *silicates* and *Na-polysulphides*, — there can only be question of a true periodically built frame-work of the whole, presumably consisting here of *Si*-, *Na*- and *O*-atoms, — in which a great number of other components are distributed in a merely statistical way.

This supposition gives an explication, on the one hand, of the particular “mobility” of a part of the *Na*-atoms and of their being replaceable by other atoms, — even by *Ca* (as in *hauyne*), which as a consequence of its bivalency, must substitute 2 *Na*-atoms at the same time. Otherwise it would be quite inexplicable in the last case, that the *Ca*-atom could substitute 2 *Na*-atoms of a rigorously periodical structure, without that structure being influenced in the least respect by that substitution, as we have experimentally demonstrated it to be the case in *hauyne* and *nosean*. The conception of these “errant” components within the fundamental lattice explains, why F. SINGER and others were able to show an intimate relation between such *zeolites* (and *permutites*) on the one hand and the *ultramarines* on the other<sup>2)</sup>; just as is the case with the  $H_2O$ -molecules in some *zeolites*, which can reversibly be deshydrated, and for

<sup>1)</sup> R. W. G. WYCKOFF, J. W. GREIG and N. L. BOWEN, Amer. Journ. Science, **11**, 459, (1926); R. W. G. WYCKOFF and G. W. MOREY, Amer. Journ. Science, **12**, 419, (1926).

<sup>2)</sup> That the ultramarines possess e.g. “hydraulic” properties, and that they are even able to increase the binding-power of cements, has first been shown by ROHLAND; vid.: Zeits. f. angew. Chemie, **17**, 609, (1904).

which an analogous supposition about their constitution now must generally be adopted. It also permits of giving in some degree an explication of the factors, on which the coloration of the ultramarines depend, and of the fact, that all these differently coloured preparations, — yellow, red, green, blue or violet, — always give almost completely *identical* diffraction-images. That colour is probably determined by the greater or smaller complexes of *S*-atoms, or of *AlS*- and *NaS*-groups, etc., and of their special degree and mode of dispersion within the total bulk of the solid mass. It is now fairly certain, that the *ultramarines* can not any longer be considered as compounds of a constant stoichiometrical composition; — a fact, to which also is pointed by the deviating results of the chemical analysis in different cases. The attribution to them of empirical formulae, — and much less that of “structural” formulae, — has no longer any significance: these substances consist of a regular, periodical structure as bearer of an irregularly distributed company of other atomic individuals, amongst which *Na*, *Al*, and *S* are the most predominant, and the number and nature of which greatly depend on the special conditions of the preparative processes. The colour of the solid mass seems to be undeniably bound to the presence of the element *sulphur*, — perhaps in a way analogous to that recently described by VON WEIMARN <sup>1)</sup>, or otherwise by the special degree of dispersion of *S*-containing components. The stability of the ultramarines at red heat, on the other hand, makes it rather improbable, that *free sulphur* <sup>2)</sup> should be the cause of their colours, as some authors have suggested.

§ 6. In the beginning of this paper it has been said, that *sodalite* (*Monte Somma*) yielded quite another type of powder-spectrogram than *nosean*, *haugne*, etc., — although formerly *sodalite* has always been considered to be a compound of a structure perfectly analogous to that of the others, *sodiumsulphate* in the latter being simply substituted by *sodiumchloride*, but in combination with the same fundamental silicate. This conception of the constitution of all the substances of this series seemed to be strongly corroborated by the results of the artificial syntheses <sup>3)</sup> of these minerals.

From the investigations of BRÖGGER and BACKSTRÖM, who place all these minerals also in one and the same group, it follows with certainty, that the symmetry of *sodalite* is surely *not hexakisoctahedral*: the corrosion-figures obtained by them were, on the contrary, in better agreement

<sup>1)</sup> P. VON WEIMARN, Koll. Chem. Beihefte, 22, 38, (1926); J. HOFFMANN, Koll. Zeits., 10, 275, (1912); F. KNAPP, DINGLER's Polyt. Journ., 233, 479, (1879); Idem 229, 74, (1878).

<sup>2)</sup> Vid. i.a.: PATERNÒ and MAZZUCHELLI, Atti R. Acad. Lincei, (V), 16, I, 465, (1907).

<sup>3)</sup> J. LEMBERG, Zeits. d. Geol. Ges., 28, 603, (1876); 35, 583, (1883); J. MOROZEWICZ, Tschem. Min. Mitt., 18, 143, (1899); O. MÜGGE, in ROSENBUSCH' „Mikrosk. Physiogr., I, 323, (1892); Z. WEYBERG, Z. Min. (1905), 653; CH. and G. FRIEDEL, Bull. Soc. min., 13, 183, (1890); S. J. THUGUTT, N. Jahrb. Min., etc., Beil. Bd. 9, 568, (1894).

with *hexacistetrahedral* symmetry. In that case, the apparently simple rhombicdodecahedra of *sodalite* must be considered to be twins parallel to  $\{100\}$ . This, indeed, is in perfect agreement with our experience in preparing rotation-spectrograms of a transparent crystal of rhombicdodecahedral shape: on rotating the crystals round  $[100]$ , the spectrograms obtained appeared to be quite normal; but on rotating them round  $[111]$ , they were completely irregular and not suited for analysis, just as it was before observed in the case of *haugyne*, — in reversed sequence, however.

In the first place in Table III are reviewed the data, relating to the powder-spectrogram of *sodalite*; the values of  $\sin^2 \frac{\theta}{2}$  are calculated

N <sup>o</sup> . of the Lines:	Intensities estimated:	2 l in mm.:	Wave-length:	$\frac{\theta}{2}$ :	$\sin^2 \frac{\theta}{2}$ : (observed):	$\sin^2 \frac{\theta}{2}$ : (calculated):	Indices $\{h\ k\ l\}$ :
1	1	23.1	$\lambda_{\beta}$	13°14'	0.0524	0.0503	(220)
2	8	25.5	$\lambda_{\alpha}$	14°37'	0.0637	0.0608	(220)
3	2	31.2	$\lambda_{\alpha}$	17°53'	0.0943	0.0912	(222)
4	3	34.3	$\lambda_{\alpha}$	19°39'	0.1131	0.1064 or 0.1216	(321) or (400)
5	3	37.5	$\lambda_{\alpha}$	21°29'	0.1341	0.1368	(330)
6	3	41.3	$\lambda_{\alpha}$	23°40'	0.1611	0.1596	(421)
7	3	42.3	$\lambda_{\alpha}$	24°14'	0.1685	0.1672	(332)
8	1	50.1	$\lambda_{\alpha}$	28°43'	0.2309	0.2280	(521)
9	1	53.0	$\lambda_{\alpha}$	30°25'	0.2563	0.2584	(530)
10	1	56.3	$\lambda_{\alpha}$	32°16'	0.2850	0.2812 or 0.2888	(601) or (611)
Radius of camera: 25 mm. $V=55000$ Volts; time of exposure: 3 hours.							
Quadratic equation: $\sin^2 \frac{\theta}{2} = 0.0076 (h^2 + k^2 + l^2)$ for $\alpha$ -radiation of 1.540 A.U.							
From this $a_0$ calculated to be 8.81 A.U.							

from the coefficient of the quadratic equation, as this was deduced from the numerical results of the BRAGG-spectrogram to be described later-on. Among the indices of Table III such symbols as:  $\{321\}$ ,  $\{421\}$ ,  $\{332\}$ ,  $\{521\}$ , etc. are present, for which, therefore,  $(h+k)$ , or  $(h+k)$ , or  $(h+l)$  are *odd*. Moreover, there are also symbols present, for which  $(h+k+l)$  is *odd*. From all these facts it must be concluded, that the fundamental

BRAVAIS' lattice of the structure of *sodalite* is the *simple cubic* lattice, — as contrasted with what was found in the case of *nosean* and *haugyne*.

§ 7. These conclusions, now, were more particularly corroborated by the analysis of a number of rotation-spectrograms and of such obtained after BRAGG's method.

A BRAGG's spectrogram on a face of  $\{110\}$ , in combination with one on  $\{100\}$  of *rocksalt* at the same distance, for the purpose of comparison, gave the value: 8.81 A.U. for  $a_0 = d_{110} \cdot \sqrt{2}$ ; from this the coefficient of the quadratic equation for  $\sin^2 \frac{\theta}{2}$  is calculated to be: 0.0076, which value was already formerly used in calculating the data of the powder-spectrogram.

A rotation-spectrogram obtained by turning the crystal round a principal axis  $[001]$ , gave the results, which are recorded in Table IV (see next page).

The distance between the principal spectrum and the first hyperbola being 7.3 mm.,  $tg \mu$  was equal to 0.1791; from which follows:  $\mu = 10^\circ 11'$ , and for  $I_{[100]}$ , — being the identity-distance in the direction of the principal axis:  $I_{[100]} = \frac{\lambda_\infty}{\sin \mu} = 8.72$  A.U., — which is in perfect accordance with the exact value: 8.81 A.U., calculated from the BRAGG-spectrogram at  $(110)$ .

For the number  $n$  of atomic complexes of the formula:  $Na_4 Al_3 Si_3 O_{12} Cl$ , which are present per fundamental cell, we find, as the specific weight  $d$  is 2.30 here:

$$n = \frac{2.30 \times (8.81)^3}{485 \times 166} = 1.96 \text{ (i.e. } = 2\text{)}.$$

Thus in each elementary cell a total mass is present, whose composition in the simplest case is expressed by:  $Na_8 Al_6 Si_6 O_{24} Cl_2$ . If the strongly deviating data of the analysis of different *sodalites* be taken into account, as they are given in literature, the question may be raised, whether also in this case there can be properly spoken of a rational stoichiometrical composition or not? Nor is, in this case, the possibility by no means excluded, that there is only question of a chemical composition, oscillating within rather wide limits.

§ 8. If the *hexacistetrahedral* symmetry of *sodalite*, as suggested by BRÖGGER and BÄCKSTRÖM, be adopted, only two space-groups:  $T_d^1$  and  $T_d^4$  need be considered here, because the fundamental lattice was proved to be the simple cubic one. But in ROENTGEN-analysis the spacings of net-planes with the symbol  $\{hhl\}$ , in which  $l$  is an *odd* number, will in  $T_d^4$  always appear to be halved<sup>1)</sup>. With absolute certainty, however, a spot  $(221)$  has been found on the rotation-spectrogram, although its

<sup>1)</sup> W. T. ASTBURY and K. YARDLEY, loco cit., p. 255.



TABLE IV.

Rotation-spectrogram of Sodalite from Monte Somma.

(Distance between plate and cristal: 40.5 mm.)

The axis of revolution here was the principal axis [001].

Distances of spots from the centre in mm.:	$\theta/2$ :	$\sin^2 \theta/2$ (observed):	$\sin^2 \theta/2$ (calculated):	Indices $\{h k l\}$ :	Sequence of hyperbolae:
25.3	16° 0'	0.0760	0.0760	(310)	Principal spectrum.
32.0	19° 9'	0.1076	0.1131	(330) $\lambda_3$	
38.9	21° 56'	0.1395	0.1368	(330) $\lambda_{\infty}$	
42.0	23° 1'	0.1529	0.1520	(420)	
52.0	26° 3'	0.1927	0.1900	(500) of (430)	
64.1	28° 51'	0.2328	0.2204	(520)	
70.0	29° 59'	0.2496	0.2432	(440)	I.
18.7	12° 24'	0.0461	0.0456	(211)	
31.5	18° 56'	0.1053	0.1064	(321)	
39.1	22° 0'	0.1402	0.1368	(411)	
52.0	26° 3'	0.1928	0.1976	(501) or (431)	II.
18.7	12° 24'	0.0461	0.0456	(112)	
24.0	15° 9 1/2'	0.0684	0.0699	(212)	
28.0	17° 20'	0.0888	0.0912	(222)	
32.1	19° 12'	0.1081	0.1064	(312)	
39.0	23° 41'	0.1613	0.1596	(412)	
48.0	24° 55'	0.1775	0.1824	(422)	
62.5	28° 32'	0.2282	0.2204	(432)	
75.0	30° 49'	0.2625	0.2508	(522)	
82.0	31° 51'	0.2785	0.2736	(442)	III.
25.8	16° 15'	0.0783	0.0760	(103)	
31.8	19° 41'	0.1067	0.1064	(123)	
37.9	21° 33'	0.1348	0.1368	(303)	
45.6	24° 12'	0.1682	0.1672	(233)	
72.0	30° 20'	0.2553	0.2584	(433) or (503)	IV.
48.6	25° 6'	0.1799	0.1824	(224)	
55.8	27° 1'	0.2063	0.2010	(314)	
77.9	31° 16'	0.2694	0.2628	(334)	

Wave-length:  $\lambda_{cu} = 1.540$  A.U.;  $V = 55000$  Volts; time of exposure: 5 hours.Quadratic Equation:  $\sin^2 \theta/2 = 0.0076 (h^2 + k^2 + l^2)$ . Accordingly:  $a_0 = 8.81$  A.U.

intensity was relatively weak. For that reason *sodalite* must necessarily <sup>1)</sup> belong to the space-group  $T_d^1$ .

In this space-group  $T_d^1$  there occur *no twofold* places; but there are *two* different kinds of *sixfold* places, the coordinates of which all contain, however, a variable parameter  $u$ . Now evidently  $Si_6$ , as well as  $Al_6$ , can both be placed in those localities; so that, in contrast with the case of *nosean*, it is *not* absolutely certain here, whether also in *sodalite* such "errant" constituents must be supposed to be present in its structure, distributed in it at random in all directions. But even then, the empirical, coordinative or structural formulae, as occasionally proposed for *sodalite*, cannot be considered as being in real agreement with the different kinds of available places in its elementary cell.

§ 9. Some intensely coloured, dull blue and opaque individuals of *lazurite* from *Afghanistan*, which superficially had the appearance of rhombic-dodecahedra, were used for comparison with the substances hitherto investigated. However, on the apparent  $\{110\}$ -faces no BRAGG-spectrograms whatsoever could be obtained, notwithstanding several endeavours were made. Evidently the so-called "crystals" were no true individuals, but probably conglomerates having only an approximate resemblance with a rounded rhombicdodecahedron. Moreover, the blue masses were very inhomogeneous and they contained a number of alien and incrustated substances, i.a. *pyrite*.

A small piece of this blue *lazurite*, as homogeneous as possibly, could be separated out of the inner parts of the mass; it was finely powdered now and used for the preparation of a powder-spectrogram after HULL-DEBIJE's method. The photograph obtained after five hours exposure, was rather weak, but the lines were sharp enough to be localised by means of a comparator. It was found, that by far the greater number of the diffraction-lines coincided with those of *hauyne* and the *ultramarines*; more especially the spectrogram of *lazurite* showed the following lines of those substances (numbers as in Table I): no's 2, 3, 4, 5, 6, 7, 9, 10, 11 and 13, while no's 8 and 12 were absent. Moreover, three very weak lines were observed, which must correspond to other constituents; although two of them rather exactly coincided with the lines no's 7 and 10 of *sodalite* (Table III), it is yet highly problematic, whether they can be attributed to that mineral, because the most conspicuous and intense line of *sodalite*, no. 2 of Table III, was certainly *not* present. There are undoubtedly other substances present in the blue mass as accessory products; but *it is quite certain now, that the spectrogram of lazurite is chiefly identical with that of hauyne and of the artificial ultramarines*. This result

<sup>1)</sup> TH. BARTH, Norsk Geol. Tidsskr., 9, 40, (1926). The paper published by this author appeared, when the structure of *sodalite* had already been determined by us. Mr. BARTH was, moreover, not able to fix his choice between the two space-groups possible here: this problem is now settled here definitely.

is in partial agreement with BRÖGGER and BÄCKSTRÖM's supposition of *lazurite* being a mixture of *haüyne*-substance, natural *ultramarine* and other accompanying substances: but their suspicion, that *sodalite* would be the accessory mineral, is not confirmed beyond all doubt by the results described in this paper. As *sodalite* and *haüyne* have different structures and, most probably, also different symmetries, — it is quite improbable, that *lazurite* can be considered as a kind of mixed crystal of isomorphous components, as *haüyne* and *sodalite* by the Swedish authors are evidently and erroneously supposed to be.

§ 10. Finally we have kept in view the question, in how far there may exist any relation between the constitution of the *ultramarines* and *nephelite*, the composition of which is in the simplest case expressed by the formula:  $Na Al Si O_4$ . *Nephelite* itself is hexagonal-pyramidal; its axial ratio, according to VON BAUMHAUER, is:  $a:c = 1:0,8314$ . That there is really some relation between the so-called „ultramarine-base” and a *sodium-aluminium-silicate* of the composition as given above, is acknowledged by all authors, who have occupied themselves with the synthesis of the *ultramarines*. BRÖGGER and BÄCKSTRÖM<sup>1)</sup> indicate, how by a moderate heating during six hours of *kaoline* with somewhat more than the theoretical quantity of  $Na_2CO_3$ , a silicate of the composition mentioned is obtained, which would be *optically-isotropic*. But, besides the hexagonal form of *nephelite* already mentioned, hitherto only a single modification of it is known: the *carnegiüte*, which is stable only at much higher temperatures<sup>2)</sup>, and which is certainly not cubic. A preparation of the composition:  $Na Al Si O_4$ , obtained by repeatedly sintering at  $900^\circ C$ . of a mixture of  $2 Si O_2$ ,  $Na_2 CO_3$  and  $Al_2 O_3$  or of  $Na Al O_2$  and  $Si O_2$ , and homogenizing the product, appeared to be identical with natural *nephelite*: the powder-spectrogram was in every respect the same as that of natural *nephelite* from the *Monte Somma, Vesuvius*.

The crystals of this *nephelite* were prismatic, with  $\{10\bar{1}0\}$  and  $\{0001\}$ , and occasionally with the pyramid  $\{1011\}$ : according to VON BAUMHAUER, they are twins or double twins after  $\{10\bar{1}0\}$  and  $\{0001\}$ , as e.g. becomes clear from the lack of symmetry and the special shape of the corrosion-figures on  $\{10\bar{1}0\}$ . This twin-formation will, however, not disturb the rotation-spectrograms, obtained by turning the crystal round an axis  $[0001]$ , as has already been discussed.

In such a spectrogram, made with a cylindrical film, the distance between the principal spectrum and the top of the first hyperbola, was found to be: 9.5 mm., the distance between photographic plate and crystal being 50 mm.: therefore,  $tg \mu = 0,190$ , or  $\mu = 10^\circ 46'$ , from which follows:  $I_{[0001]} = 8,3 A.U.^2$ ).

<sup>1)</sup> W. C. BRÖGGER en H. BÄCKSTRÖM, Zeits. f. Kryst., 18, 242, (1891).

<sup>2)</sup> During the correction of this proof the said rotation-spectrogram was made with a new cylindrical camera and an intensifying screen. The new measurements made it necessary to improve the numbers of the Dutch publication.

This dimension could be determined more exactly by means of a BRAGG-spectrogram on (0001),  $d_{(0001)}$  being found to be equal to:  $4.19 \text{ A.U.} = \frac{1}{2} \times 8.38 \text{ A.U.}$  The dimension of the hexagonal cell in the direction of the  $c$ -axis is, therefore, equal to:  $8.38 \text{ A.U.}$ , this spacing being halved apparently in ROENTGEN-analysis. Of all space-groups in the hexagonal system, where this can occur, only  $C_6^2$  can be attributed to this mineral of hexagonal-pyramidal symmetry.

On rotating the crystal round a perpendicular to (1010), a rotation-spectrogram was obtained, from which the identity-distance in that direction was deduced to be:  $16.75 \text{ A.U.}$  For the identity-distance in the direction of the  $a$ -axis, therefore, the value:  $9.67 \text{ A.U.}$  is obtained. A BRAGG-spectrogram on (1010) showing two diffraction-lines, gave however:  $d_{(10\bar{1}0)} = 2.85 \text{ A.U.}$  and  $4.28 \text{ A.U.}$  respectively; as:  $3 \times 2.85 = 2 \times 4.28 = 8.56 \text{ A.U.}$ , the diffraction-images observed here are evidently of the 3<sup>rd</sup>, respectively of the 2<sup>nd</sup> order. No reflexion of the 1<sup>st</sup> order was ever observed; and for the dimension of the cell in the direction of the  $a$ -axis, the exact value:  $9.87 \text{ A.U.}$  is immediately deduced. Thus the axial ratio  $a : c$  is equal to:  $9.87 : 8.38 = 1 : 0.849$ ; it appears, therefore, that to the form  $\{10\bar{1}1\}$  in VON BAUMHAUER's measurements, really the symbol:  $\{10\bar{1}1\}$  must be given.

As the specific gravity of the *nephelite* is: 2.62, the number  $n$  of the "molecules" of the composition:  $\text{NaAlSiO}_4$ , which are present within a hexagonal cell (i.e. in  $\frac{1}{3}$  of the total combination of six triangular prisms), is calculated to be 8. In each triangular prism, therefore, the total mass present, must be written as:  $\text{Na}_4\text{Al}_4\text{Si}_4\text{O}_{16}$ . In the space-group  $C_6^2$  only two- and sixfold places, the last ones with three variable parameters, — can be present; as the number of atoms per cell is so great and the values of the dimensions in the directions  $a$  and  $c$  are also very considerable, — it may be considered impossible to determine these parameters for all atoms present. The same is true for the powder-spectrogram obtained with the artificial silicate prepared from  $2 \text{ Si O}_2$ ,  $\text{Al}_2\text{O}_3$  and  $\text{Na}_2\text{CO}_3$ , mentioned above. Of course, in no case can there be question to state already now a closer relation of this silicate with the ultramarines.

Whether it will appear to be possible to ascertain completely, which constituents form the rigid crystal-skeleton of the ultramarines and which are the "errant" ones, can only be decided by further experiments, which are already in progress.

*Groningen, Laboratory for Inorganic and Physical  
Chemistry of the State-University.*

**Mathematics.** — *A complex of Conics.* By Prof. JAN DE VRIES.

(Communicated at the meeting of Januari 29, 1927).

The conics  $k^2$  cutting the fixed conic  $a^2$  in the plane  $a$  twice and resting on the lines  $b_1, b_2, b_3$ , form a *complex*,  $\Gamma$ .

1. *Pairs of lines in  $\Gamma$ .* (a.) Any transversal  $t_{123}$  of the lines  $b$  forms a  $k^2$  with any line  $r$  cutting it in the plane  $a$ ; to any  $r$  there correspond two  $t_{123}$ .

(b.) Any transversal  $t_{12}$  of  $b_1, b_2$  and  $a^2$  forms a  $k^2$  with any line  $r$  that rests on  $t_{12}, b_3$  and  $a^2$ . The lines  $t_{12}$  form a scroll of the fourth degree; the lines  $r$  belong to a congruence [2,2]; any  $r$  defines three  $t_{12}$ .

(c.) Each of the four transversals  $a$  of  $a^2, b_1, b_2, b_3$ , is completed to a  $k^2$  by any ray  $r$  of the congruence [1,2] that has  $a$  and  $a^2$  as directrices.

(d.) The line  $a_{12}$  lying in  $a$  and resting on  $b_1$  and  $b_2$  forms a  $k^2$  with any line  $r$ , that cuts  $a_{12}$  and  $b_3$ .

(e.) Any line  $r_{12}$  resting on  $b_1$  and  $b_2$  is completed to a  $k^2$  by the line  $r$  in  $a$  that cuts  $r_{12}$  and  $b_3$ .

Evidently each of (b), (d) and (e) represent three different systems.

2. *Conics through a given point.* The  $k^2$  through a point  $C$  form a surface  $\Psi$ ; in order to determine the degree we shall investigate the intersection with the plane  $a$ .

Any plane through  $C$  and a point  $A$  of  $a^2$  contains a  $k^2$ ; these  $k^2$  form a dimonoid of the fourth degree which has five points  $A^*$  outside  $A$  in common with  $a^2$ , hence  $a^2$  is a *fivefold curve* on  $\Psi$ . Through  $C$  there passes a transversal  $r$  of  $a_{kl}$  and  $b_m$ ; accordingly  $\Psi$  contains the lines  $a_{12}, a_{23}, a_{31}$ . The transversal of  $b_k$  and  $b_l$  through  $C$  yields a line of  $a$  that cuts it and  $b_m$ .

The intersection of  $\Psi$  and  $a$  consists, therefore, of the five-fold  $a^2$  and six straight lines; consequently the surface is of the degree 16.

Its intersection with the plane  $Cb_1$  consists of the following figures. In the first place a  $k^2$  cutting  $b_1$  twice, which must, therefore, be counted double. Through  $C$  there pass two lines  $r$  cutting  $b_1$  and  $a^2$  each of which defines three lines  $t_{23}$  and is, accordingly, a triple line of  $\Psi$ . Further  $Cb_1$  contains three lines as component parts of pairs of lines of which one line lies in  $a$  (§ 1, d and e). And now it appears that  $b_1$  must be a *triple line*.

This may be confirmed in the following way. The conics through  $C$



and a point of  $b_1$  which cut  $a^2$  twice and rest on  $b_2$ , form a cubic dimonoid; this contains, therefore, three  $k^2$  that also cut  $b_3$ .

Now the intersection with  $Cb_k$  shows that  $C$  is an *eleven-fold* point of  $\Psi^{16}$ .

$\Psi$  contains the four transversals of  $a^2$ ,  $b_1$ ,  $b_2$ ,  $b_3$ , and the corresponding lines  $r$  through  $C$ . Further the 18 lines  $t_{kl}$  corresponding to the 6 above mentioned triple lines  $r$ .

3. Surface of the  $k^2$  that cut a given line  $l$  twice. The  $k^2$  that cut  $l$  in the point  $L$ , form a  $\Psi^{16}$  with an 11-fold point  $L$ ; hence there are 5  $k^2$  which cut  $l$  in another point  $L^*$ . As each plane through  $l$  contains one  $k^2$ , the said surface is of the degree *seven*.

Its intersection with  $a$  consists of  $a^2$  and five lines three of which rest on  $l$  and a line  $b$  whereas the other two cut  $l$  and a transversal of  $l$  and the three lines  $b_k$ . The lines  $b$  are single on  $\Phi^7$ .

There are ten  $k^2$  that touch the line  $l$ .

4. Surface of the  $k^2$  that cut the lines  $b$  in projective point ranges. The planes of these  $k^2$  osculate a twisted cubic;  $a^2$  is, therefore, a triple conic of this surface  $\Omega$ . The intersection of  $b_k$  and  $a$  defines a pair of lines of which one lies in  $a$  and the other one rests on  $b_l$  and  $b_m$ . Consequently the degree of  $\Omega$  is *nine*.

The scroll defined by two of the point ranges cuts  $a^2$  four times; hence  $\Omega^9$  contains twelve pairs of lines of which one of the lines rests on two  $b_k$ , the other one on the third line  $b$ .

5. Surface  $\Lambda$  of the  $k^2$  that cut two more lines,  $b_4$  and  $b_5$ . The  $k^2$  of  $\Gamma$  that cut  $b_4$  in a point  $B_4$ , form a surface of the degree 16 (§ 2); accordingly there are 16  $k^2$  which also rest on  $b_5$ . Hence on the surface  $\Lambda$  the 5 lines  $b$  are 16-fold.

$a$  contains two lines that cut a line  $b$  and one of the two transversals of the other four lines  $b$ . Further  $a$  contains the lines which each cut two lines  $b$  and rest on a transversal of the other three lines  $b$  and which are, accordingly, double lines. Consequently in all 10 lines and 10 double lines of  $\Lambda$  lie in  $a$ .

The  $k^2$  through a point  $A$  of  $a^2$  resting on the 5 lines  $b$ , form a surface  $\Phi$  of the degree 18 ( $P\nu^6=18$ ). Its intersection with the plane  $Ab_5$  consists of a  $k^2$  that cuts  $b_5$  twice and is, accordingly, a double curve, the quadruple line  $b_5$  ( $P^2\nu^4=4$ ), two lines through  $A$  that cut a transversal of  $b_1$ ,  $b_2$ ,  $b_3$  and  $b_4$ , and four double lines through  $A$  each of which cuts one of the lines  $b$  and two transversals of the other three. Accordingly a straight line through  $A$  in  $Ab_5$ , has 6 points outside  $A$  in common with  $\Phi^{18}$ . As this is also the case in each of the other planes  $Ab_k$ ,  $A$  is a 12-fold point of  $\Phi$ ; hence  $a^2$  and  $\Phi^{18}$  have 24 points outside  $A$  in common and  $a^2$  is a 24-fold curve of  $\Lambda$ .

Finally  $a$  contains the  $k^2$  which cuts the 5 lines  $b$  and which apparently must be counted six times. The entire intersection of  $A$  and  $a$  is, therefore, of the order  $10 + 20 + 48 + 12$ ; hence  $A$  is a surface of the degree 90.

On  $A$  there lie  $10 \times 4 \times 3$  pairs of lines  $(r, r')$ , of which  $r$  rests on  $a^2$  and  $3 b_k$ ,  $r'$  on  $a^2$ ,  $r$ , and the other 2  $b$ .

The surface  $\Phi^7$  of the  $k^2$  of  $\Gamma$  that have  $b_4$  as chord, has 7 points in common with  $b_5$ . Hence  $A$  has besides  $5 \times 7 = 35$  double curves  $k^2$  the planes of which pass through one of the lines  $b$ .

6. Consequently there are 90 conics that cut a given conic twice and that rest on six given lines.

If we use the symbol  $k$  to indicate that a conic rests on a given conic, we may express the result found above by  $k^2 \gamma^6 = 90$ . This number (and other ones containing  $k$ ) can be found in the following way by applying the principle of the conservation of the number.

In order to determine  $P^2 k^2 \gamma^2$  we place one of the lines,  $b_1$ , in the plane of the given conic  $a^2$ . The plane through the points  $P_1$ ,  $P_2$  and one of the points of intersection of  $b_1$  and  $a^2$  contains one  $k^2$  which satisfies the conditions. The same also holds good for the configuration of  $P_1 P_2$  and the line of  $a$  resting on it and on  $b_2$ . Hence  $P^2 k^2 \gamma^2 = 3$  (cf. § 2).

In order to determine  $P k^2 \gamma^4$  we choose three lines  $b_1, b_2, b_3$  in a plane  $\varphi^1$ ). In this case there satisfy in the first place the  $3 \times 3$  figures  $k^2$  through  $P$  and one of the points  $b_1 b_2, b_1 b_3, b_2 b_3$  which rest twice on  $a^2$  and also on  $b_4$ . Further 7 pairs of lines  $(r, r')$  of which  $r$  lies in  $\varphi$  and is a chord of  $a^2$  or rests on  $a^2$  and  $b_4$  or cuts  $a^2$  and a transversal  $r'$  of  $a^2$  and  $b_4$ . Consequently  $P k^2 \gamma^4 = 16$  (cf. § 2).

In order to find  $k^2 \gamma^6$  we again choose  $b_1, b_2, b_3$  in  $\varphi$ . In this case there satisfy in the first place the  $3 \times 16$   $k^2$  through one of the points  $b_1 b_2, b_1 b_3$  or  $b_2 b_3$ , which rest on the other four  $b_k$  and cut  $a^2$  twice. Further the  $k^2$  in  $\varphi$  that cuts  $a^2$  twice and rests on  $b_4, b_5, b_6$ ; evidently this must be counted eight times.

There are three chords of  $a^2$  each of which is completed to a  $k^2$  by a line of  $\varphi$  which cuts one of the lines  $b_4, b_5, b_6$  and three chords of  $a^2$  to which there belongs a transversal in  $\varphi$  of two of these lines  $b$ .

Each of the six lines  $r$  of  $\varphi$  that rest on  $a^2$  and one of the lines  $b_4, b_5, b_6$  is completed to pairs of lines by three transversals of  $r$ ,  $a^2$  and the other two of these lines  $b$ . The chord of  $a^2$  in  $\varphi$  belongs to two pairs of lines. Finally each of the four transversals of  $a^2, b_4, b_5, b_6$  forms a pair of lines with the line that joins its intersection with  $\varphi$  to one of the intersection with  $a^2$ .

<sup>1)</sup> In this way I have again determined some time ago the known numbers  $P \gamma^6 = 18$  and  $\gamma^8 = 92$ . (These Proceedings, 4, 181).

Consequently we find in all  $48 + 8 + 3 + 3 + 18 + 2 + 8 = 90$  figures; hence  $k^2\nu^6 = 90$ .

7. A plane  $\varrho$  through  $b_1$  has also a curve of the order 74 in common with the surface  $\mathcal{A}^{90}$ . This cuts  $b_1$  in the first place in the  $7 \times 2$  points of intersection with the  $k^2$ , that have  $b_1$  as chord. In each of the remaining 60 points of intersection the plane  $\varrho$  is touched by a  $k^2$  of  $\mathcal{I}$ . Hence the locus of the points of contact of conics  $k^2$  with  $\varrho$  is a curve of the order 60 and the tangent  $k^2$  form a surface of the 120<sup>th</sup> degree. Hence  $k^2\nu^5\varrho = 120$ .

Applying a similar reasoning to the surface  $\Psi^{16}$  (§ 2) we find the number  $Pk^2\nu^3\varrho = 22$ .

Also these results are easily verified by the method of § 6.

In the first place we find  $P^2k^2\nu\varrho = 4$  by remarking that any plane through  $P_1P_2$  contains two conics which cut  $\alpha^2$  twice, touch the plane  $\varrho$  and pass through  $P_1$  and  $P_2$ .

In order to arrive at  $Pk^2\nu^3\varrho$  we again choose the three lines  $b$  in a plane  $\varphi$ . In this case there satisfy  $3 \times 4$   $k^2$  through  $P$  and a point  $b_kb_l$ . Further the chord of  $\alpha^2$  in  $\varphi$  belongs to a pair of lines that must be counted twice. Finally there are four pairs of lines  $(r, r')$  to be counted twice, with a double point on  $\varphi\varrho$  of which  $r$  passes through  $P$  and  $r'$  lies in  $\varphi$ . Hence  $Pk^2\nu^3\varrho = 22$ .

The number  $k^2\nu^5\varrho$  is found in the following way.  $\varphi$  contains two conics each of which must be counted eight times. Through each point  $b_kb_l$  in  $\varphi$  there pass 22  $k^2$ . The chord of  $\alpha^2$  in  $\varphi$  belongs to a pair of lines with double point on  $\varphi\varrho$ . There are eight pairs  $(r, r')$  of which  $r$  lies in  $\varphi$  and rests on  $\alpha^2$  and  $b_4$  or  $b_5$  and the double point lies on  $\varphi\varrho$ . Further there are eight pairs of which  $r'$  rests on  $\alpha^2$ ,  $b_4$ ,  $b_5$  and  $\varphi\varrho$  and  $r$  lies in  $\varphi$ . Finally the two pairs  $(r, r')$  satisfy of which  $r'$  is a chord of  $\alpha^2$  and cuts the lines  $b_4$  ( $b_5$ ) and  $\varphi\varrho$ . Accordingly  $16 + 66 + 2 + 16 + 16 + 4 = 120$ .

---

**Geology.** — *Fractures and faults near the surface of moving geanticlines. IV. The elastic rebound of the earth crust in Central Honshu.* By Prof. H. A. BROUWER.

(Communicated at the meeting of January 29, 1927).

During the 3<sup>d</sup> Pan Pacific Science Congress (October, November 1926) some of the members including myself were so fortunate to visit the regions that were devastated by the Mino-Owari earthquake of 1891 in Central Japan, guided by Prof. S. NAKAMURA of the University of Kyoto. This part of Central Japan is known for its numerous and important fractures and faults with horizontal and vertical displacements, formed during this earthquake, and which are clearly visible in the topography. The fault-scarp near Midori, which cuts a new road in two, showing clearly a vertical displacement of 6 m. and a horizontal one of 4 m., is reproduced in many handbooks as a typical example of movement along fractures during earthquakes.

Of this earthquake an extensive description is given by KOTO<sup>1)</sup>. He describes a fracture which can be followed over a distance of 112 km. from the Kisogawa to Fukui with striking regularity and clearness right through the mountains, valleys and plains of Central-Japan.

The origin of this great transverse fracture is of importance in connection with my views exposed a few years ago, about the relation between earthquakes and the movement of geanticlines in space<sup>2) 3)</sup>. It is clear, that such a regular and sharply defined transverse fracture, which can be followed nearly over the whole width of Central-Japan cannot be connected with local deformations and differences of velocity, but that here the deformation of the whole island Honshu has to be considered.

So we find a confirmation of my thesis, that *important transverse fractures near the surface of moving geanticlines will occur near the bending points of the horizontal projection of the geanticlinal axes*<sup>4)</sup>.

<sup>1)</sup> B. KOTO. On the cause of the great earthquake in Central-Japan. 1891. Journ. Coll. of Science. Imp. Univ. Japan. Vol. 5. Part IV, p. 295. Tokyo. 1893.

<sup>2)</sup> H. A. BROUWER. Some relations of earthquakes to geological structure in the East-Indian archipelago. Bull. Seism. Soc. of America. 11, 1921, blz. 166.

<sup>3)</sup> H. A. BROUWER. The geology of the Netherlands East-Indies. University of Michigan Studies. Scient. Ser. Vol. 3. p. 84—92.

<sup>4)</sup> H. A. BROUWER. The horizontal movement of geanticlines and the fractures near their surface. Journ. of Geology. Vol. 29, 1921, p. 576.

Fig. 1 gives a schematic geanticlinal axis of the island Honshu and it is evident, that the great transverse fracture along which movement took place during the Mino-Owari earthquake intersects the island near this point. Thus follows, that the deformation and movement of the island Honshu take place in a way which are incidental to important differences in velocity in a horizontal direction f.i. near the fracture in the central part of the island. These slow deformations which take place continually, cause a strain in the crust which is occasionally relieved by sudden and rapid movements along the fractures, causing the earthquakes.<sup>1)</sup>

Fig. 1. Schematic representation of the geanticlinal axis of the island Honshu (2) and the fracture of the Mino-Owari earthquake of 1891 near the bending point of the geanticlinal axis (1).



The strength of the rocks will generally be exceeded in the first place where the differences in velocity of the slow movements are the greatest.

The displacements are shown in fig. 2.

During an earthquake the crust suddenly springs back to positions of equilibrium as shown by the arrows 1 in fig. 2.

In applying this theory to the movements during the earthquake of 1891 in Central-Japan we find, that in contradiction to what could be concluded from the above mentioned considerations, the N.E. part along



Fig. 2. Relative displacements along the fracture of the Mino-Owari earthquake. 1, derived from the shape of the geanticlinal axis of the island Honshu. 2, the observed relative displacements.

<sup>1)</sup> H. F. REID. The elastic rebound theory of earthquakes. University of Calif. Publ. Bull. Dept. Geol., Vol. 6. N<sup>o</sup>. 19, 1911.



the fracture has been moved to the N.W., with regard to the S.W. part (2 in Fig. 2).

It could be concluded, that the movement of 1891 was an exception to the general rule, but in the following it will be shown that this needs not be the case and that the displacement at the surface can be in opposite direction of that at greater depth, the shape at the surface of the geanticlines being in the first place dependent on the movement at greater depths.

We did formerly already point out <sup>1)</sup> the existence of tectonic zones and zones of movement at different depths. The velocity of horizontal movement at greater depth can show an important difference with that near the surface and if the horizontal movement is considered during a sufficiently long period the result will be, that points which in the beginning of the movement were situated on a vertical line, afterwards are on an irregular curve.

The deformation and the shear, tension and compression will change from place to place. In fig. 3 we have supposed for the sake of simplicity that of two adjacent parts of the crust one part deforms but slightly. In the other part the forces cause bending as well as compression. In the upper part, that is in the part nearer to the surface, the lengthening by bending is greater than the shortening by compression; in the lower

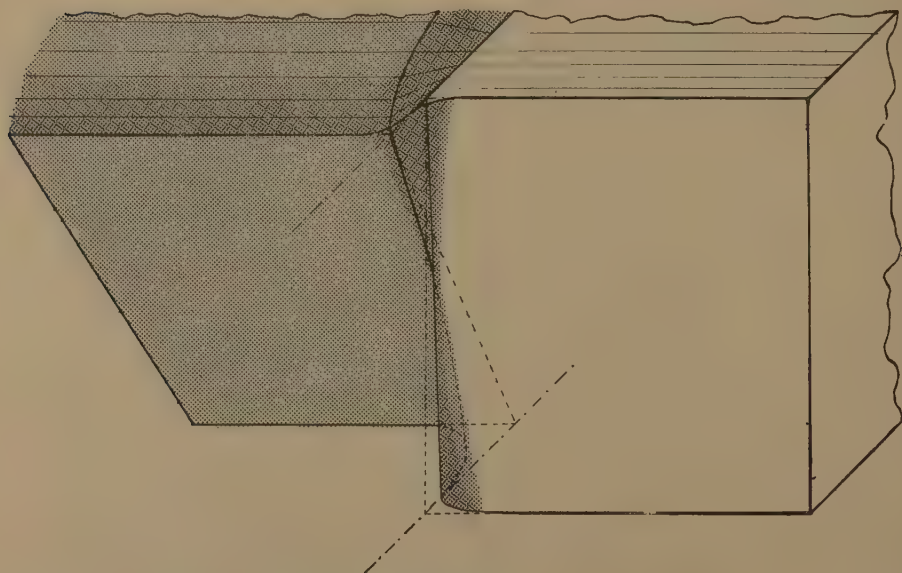


Fig. 3.

<sup>1)</sup> H. A. BROUWER. Fractures and Faults near the surface of moving geanticlines. II. Abnormal strikes near the bending-points of the geanticlines. *Proceed. Kon. Akad. v. Wet. Amsterdam*. Vol. 25, Nos 7 and 8, 1922.

part, that is in the part farther from the surface, shortening is due to bending as well as to compression.

When the strength is exceeded a sudden movement takes place along a fracture plane as is shown in exaggerated proportion in fig. 3. This movement causes a variable vertical displacement, the horizontal displacement in the upper part will have a direction *opposite* to that of the part situated farther from the surface.

The dimensions of the above mentioned parts of the crust can be left unconsidered. The distribution of the stress can vary very considerably for different depths. All this shows, that with a certain distribution of stress the movement described above can occur and that it is not permitted to make conclusions with regard to the direction of movement based on relative displacements at the surface.

When the horizontal projection of a geanticlinal axis shows a bending point, this proves differences of velocity at greater depths. The movements may be directed differently at both sides of the bending point, they can, however, also have the same direction but with a difference of velocity.

The strain near the bending point will differ from place to place and the amount of horizontal and vertical displacement will also differ from place to place. But the tendency of movement, which can be derived from the shape of the geanticline permits the supposition that a certain strain will predominate over greater distances near the fracture plane, so that definite vertical and horizontal relative displacements can also predominate.

---

**Mathematics.** — *The invariants of linear connexions with different transformations.* By Prof. J. A. SCHOUTEN. (Communicated by Prof. HENDRIK DE VRIES.)

(Communicated at the meeting of February 26, 1927).

A RIEMANNIAN connexion with fundamental tensor  $g_{\lambda\mu}$  being transformed conformally

$$g'_{\lambda\mu} = \sigma g_{\lambda\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the quantity of curvature is transformed into

$$K'_{\omega\mu\lambda\nu} = K_{\omega\mu\lambda\nu} - g_{[\omega} g_{\mu] \lambda} s_{\nu]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

being

$$s_{\mu\lambda} = 2 \nabla_{\mu} s_{\lambda} - s_{\mu} s_{\lambda} + 1/2 s_{\alpha} s^{\alpha} g_{\mu\lambda} ; \quad s_{\lambda} = \partial_{\lambda} \log \sigma \quad . \quad . \quad (3)$$

Hence the connexion is then and only then conformal-euclidian if the quantity of curvature has the form

$$K_{\omega\mu\lambda\nu} = g_{[\omega} L'_{\mu] \lambda} s_{\nu]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$L'_{\mu\lambda}$  being symmetrical, and if moreover a field  $s_{\lambda}$  can be found so that

$$2 \nabla_{\mu} s_{\lambda} - s_{\mu} s_{\lambda} + 1/2 s_{\alpha} s^{\alpha} g_{\mu\lambda} = L'_{\mu\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Now it is remarkable that the conditions of integrability of this equation are a consequence of (4) because of the identity of BIANCHI. Hence the equations (4) by themselves are necessary and sufficient to secure the conformal-euclidean property of the connexion<sup>2)</sup>. In other wise we can say that the quantity

$$C_{\omega\mu\lambda}^{\dots\nu} = K_{\omega\mu\lambda}^{\dots\nu} - g_{[\mu} L_{\lambda]}^{\nu]} s_{\omega]} ; \quad L_{\mu\lambda} = -K_{\mu\lambda} + \frac{1}{2(n-1)} K g_{\mu\lambda} \quad . \quad (6)$$

is an invariant with conformal transformations<sup>3)</sup> and that the connexion is then and only then conformal-euclidean if this quantity vanishes.

WEYL<sup>4)</sup> has shown that this peculiarity occurs also in the case of the transformations of an affine connexion that leaves the geodesics invariant. Also in that case the conditions of integrability are a consequence of the algebraical conditions and the identity of BIANCHI and the connexion is then and only then projective-euclidean if an invariant quantity of the fourth degree vanishes.

<sup>1)</sup>  $\partial_{\lambda}$  stands for  $\frac{\partial}{\partial x^{\lambda}}$ .

<sup>2)</sup> Math. Zeitschr. 11 (1921), p. 104—122; R.K. (Der Ricci Kalkül. Springer 1924), p. 169.

<sup>3)</sup> WEYL, Math. Zeitschr. 2 (1918), p. 384—411; Göttinger Nachrichten (1921), p. 99—112. Comp. R.K. p. 170.

<sup>4)</sup> Göttinger Nachrichten (1921), p. 99—112. Comp. R. K., p. 131.

In the following pages will be shown that the same peculiarity occurs in two other cases.

### 1. Transformations leaving invariant all directions.

By a linear connexion with  $C_{\lambda\mu}^{\nu}=0$ <sup>1)</sup>, but not necessary a symmetrical one, the local  $E_n$  in every point  $P$  of the  $X_n$  is represented linearly on every  $E_n$  belonging to a neighbouring point. Now we transform the connexion in such a way that the representation of all directions rests the same.

The formula for a general transformation is

$${}'I_{\lambda\mu}^{\nu} = I_{\lambda\mu}^{\nu} + A_{\lambda\mu}^{\cdot\cdot\nu} \dots \dots \dots (7)$$

$A_{\lambda\mu}^{\cdot\cdot\nu}$  being an arbitrary quantity of the third degree. If a field  $v^{\nu}$  is chosen in such a way that in an arbitrary point  $P$   $\nabla_{\mu} v^{\nu}$  is zero, then the differential

$${}'\delta v^{\nu} = \delta v^{\nu} + A_{\lambda\mu}^{\cdot\cdot\nu} v^{\lambda} dx^{\mu} = A_{\lambda\mu}^{\cdot\cdot\nu} v^{\lambda} dx^{\mu} \dots \dots \dots (8)$$

must have the same direction as  $v^{\nu}$  for every choice of  $v^{\nu}$  and  $dx^{\mu}$ . This is then and only then possible when  $A_{\lambda\mu}^{\cdot\cdot\nu}$  has the form

$$A_{\lambda\mu}^{\cdot\cdot\nu} = A_{\lambda}^{\nu} p_{\mu} \dots \dots \dots (9)$$

$p_{\lambda}$  being an arbitrary vectorfield. Hence the general formula for a transformation leaving invariant the representation of directions is (7) with  $A_{\lambda\mu}^{\cdot\cdot\nu} = A_{\lambda}^{\nu} p_{\mu}$ . The general formula for the transformed quantity of curvature is

$${}'R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} = R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} - 2\nabla_{[\omega} A_{\lambda]|\mu}^{\cdot\cdot\nu} - S_{\omega\mu}^{\cdot\cdot\alpha} A_{\lambda\alpha}^{\cdot\cdot\nu} - 2A_{\omega\alpha}^{\cdot\cdot\nu} A_{\lambda|\mu}^{\cdot\cdot\alpha} \dots (10)$$

Substituting (9) in this equation we get

$$\left. \begin{aligned} {}'R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} &= R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} - 2A_{\lambda}^{\nu} \nabla_{[\omega} p_{\mu]} + 2S_{\omega\mu}^{\cdot\cdot\alpha} p_{\alpha} A_{\lambda}^{\nu} - 2A_{\lambda}^{\nu} p_{[\omega} p_{\mu]} \\ &= R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} - 2A_{\lambda}^{\nu} \partial_{[\omega} p_{\mu]} \end{aligned} \right\} \quad (11)$$

and

$${}'V_{\omega\mu} = V_{\omega\mu} - 2n \partial_{[\omega} p_{\mu]} \quad ; \quad V_{\omega\mu} = R_{\omega\mu\alpha}^{\cdot\cdot\alpha} \dots \dots (12)$$

hence

$${}'R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} - \frac{1}{n} {}'V_{\omega\mu} A_{\lambda}^{\nu} = R_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu} - \frac{1}{n} V_{\omega\mu} A_{\lambda}^{\nu} = \text{invariant.} \dots (13)$$

Writing  $A_{\omega\mu\lambda}^{\cdot\cdot\cdot\nu}$  for this invariant quantity we deduce immediately

$$A_{\omega\mu\lambda}^{\cdot\cdot\cdot\lambda} = 0 \dots \dots \dots (14)$$

and

$$A_{\alpha\mu\lambda}^{\cdot\cdot\alpha} = R_{\mu\lambda} - \frac{1}{n} V_{\lambda\mu} \dots \dots \dots (15)$$

<sup>1)</sup> R.K. p. 75.

From (15) we see that there exists also an invariant quantity of the second degree, which is in general not zero.

$A_{\omega\mu\lambda}^{\dots\nu}$  is zero if the transformed connexion is euclidean (= integrable). But vice versa, if  $A_{\omega\mu\lambda}^{\dots\nu}$  vanishes it is possible to transform the connexion into an euclidean one by a transformation of the form (7,9).

*Proof.*

The supposition  $A_{\omega\mu\lambda}^{\dots\nu} = 0$  is aequivalent to the supposition that  $R_{\omega\mu\lambda}^{\dots\nu}$  has the form

$$R_{\omega\mu\lambda}^{\dots\nu} = M_{\omega\mu} A_{\lambda}^{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

By contraction we get  $M_{\omega\mu} = \frac{1}{n} R_{\omega\mu\alpha}^{\dots\alpha}$ . Now we try to find a field  $p$ , so that

$$2 \partial_{[\omega} p_{\mu]} = M_{\omega\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The conditions of integrability are

$$\partial_{[\xi} M_{\omega\mu]} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

But by applying the identity of BIANCHI on (16) we get

$$\nabla_{[\xi} M_{\omega\mu]} A_{\lambda}^{\nu} = -S_{[\omega\mu}^{\dots\alpha} R_{\xi]}^{\dots\nu} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

from which equation follows

$$\partial_{[\xi} M_{\omega\mu]} = -S_{[\omega\xi}^{\dots\alpha} M_{\mu|\alpha]} - S_{[\mu\xi}^{\dots\alpha} M_{\omega|\alpha]} - 2 S_{[\omega\mu}^{\dots\alpha} M_{\xi]\alpha} = 0 \quad (20)$$

Hence the conditions of integrability are a consequence of (16) and the identity of BIANCHI, and we have got the following theorem:

*It is then and only then possible to transform a linear connexion (with  $C_{\omega\mu}^{\dots\nu} = 0$ ) into an euclidean one by a transformation leaving invariant all representations of directions, if the quantity of curvature has the form  $M_{\omega\mu} A_{\lambda}^{\nu}$ , in other words if the quantity  $A_{\omega\mu\lambda}^{\dots\nu} = R_{\omega\mu\lambda}^{\dots\nu} - \frac{1}{n} R_{\omega\mu\alpha}^{\dots\alpha} A_{\lambda}^{\nu}$ , which is an invariant with these transformations, vanishes.*

Besides  $A_{\omega\mu\lambda}^{\dots\nu}$  there exists another invariant quantity of the fourth degree, namely the projective quantity of curvature  $P_{\omega\mu\lambda}^{\dots\nu}$  belonging to the symmetrical connexion with the parameters

$$\overset{0}{\Gamma}_{\lambda\mu}^{\nu} = \Gamma_{(\lambda\mu)}^{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

By substituting  $A_{\lambda\mu}^{\dots\nu} = -S_{\lambda\mu}^{\dots\nu}$  in the general formula (10) it follows that the quantity of curvature of this connexion is

$$\overset{0}{R}_{\omega\mu\lambda}^{\dots\nu} = R_{\omega\mu\lambda}^{\dots\nu} + 2 \nabla_{[\omega} S_{\lambda|\mu]}^{\dots\nu} - 2 S_{\omega\mu}^{\dots\alpha} S_{\alpha}^{\dots\nu} - 2 S_{\alpha[\omega}^{\dots\nu} S_{\lambda]\mu}^{\dots\alpha} \quad . \quad (22)$$

and from  $\overset{0}{R}_{\omega\mu\lambda}^{\dots\nu}$  we get  $P_{\omega\mu\lambda}^{\dots\nu}$  in the well known way <sup>1)</sup>.

<sup>1)</sup> C.f. f.i. R.K. p. 131.



## 2. Transformations of the form $'I_{\lambda\mu}^{\nu} = I_{\lambda\mu}^{\nu} + q_{\lambda} A_{\mu}^{\nu}$ .

In the second place we consider the case where the representation of the vectors in  $P$  on the  $E_n$  in a neighbouring point  $Q$  is changed in such a way that 'all extremities of vectors move parallel to  $\delta x^{\nu}$ . If a field  $v^{\nu}$  is chosen so that  $\nabla_{\mu} v^{\nu}$  is zero in  $P$ , then  $'\delta v^{\nu}$  must have the same direction as  $dx^{\nu}$ . From this follows that  $A_{\lambda\mu}^{\nu}$  has the form

$$A_{\lambda\mu}^{\nu} = q_{\lambda} A_{\mu}^{\nu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$q_{\lambda}$  being an arbitrary vectorfield. The transformed quantity of curvature is

$$\left. \begin{aligned} 'R_{\omega\mu\lambda}^{\nu} &= R_{\omega\mu\lambda}^{\nu} - 2 A_{[\mu}^{\nu} \nabla_{\omega]} q_{\lambda} + 2 S_{\omega\mu}^{\nu} q_{\lambda} - 2 A_{[\omega}^{\nu} q_{\mu]} q_{\lambda} \\ &= R_{\omega\mu\lambda}^{\nu} + 2 A_{[\omega}^{\nu} (\nabla_{\mu]} q_{\lambda} - q_{[\mu} q_{\lambda]}) + 2 S_{\omega\mu}^{\nu} q_{\lambda} \end{aligned} \right\} \quad (24)$$

hence

$$\left. \begin{aligned} 'R_{\mu\lambda} &= R_{\mu\lambda} + n \nabla_{\mu} q_{\lambda} - \nabla_{\mu} q_{\lambda} - n q_{\mu} q_{\lambda} + q_{\mu} q_{\lambda} + S_{\alpha\mu}^{\alpha} q_{\lambda} \\ &= R_{\mu\lambda} + (n-1) (\nabla_{\mu} q_{\lambda} - q_{\mu} q_{\lambda}) + 2 S_{\alpha\mu}^{\alpha} q_{\lambda} \end{aligned} \right\} \quad (25)$$

and

$$'S_{\lambda\mu}^{\nu} = S_{\lambda\mu}^{\nu} + q_{[\lambda} A_{\mu]}^{\nu}; \quad 'S_{\alpha\mu}^{\alpha} = S_{\alpha\mu}^{\alpha} - 1/2 (n-1) q_{\mu} \quad . \quad (26)$$

From (24-26) we deduce that the quantity

$$\left. \begin{aligned} T_{\omega\mu\lambda}^{\nu} &= R_{\omega\mu\lambda}^{\nu} - \frac{2}{n-1} A_{[\omega}^{\nu} R_{\mu]\lambda} + \\ &\quad + \frac{4}{n-1} S_{\omega\mu}^{\nu} S_{\alpha\lambda}^{\nu} - \frac{8}{(n-1)^2} A_{[\omega}^{\nu} S_{|\alpha|\mu]}^{\alpha} S_{\beta\lambda}^{\nu} \end{aligned} \right\} \quad (27)$$

is invariant. From (27) follows

$$T_{\alpha\mu\lambda}^{\alpha} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

and

$$T_{\omega\mu\alpha}^{\alpha} = V_{\omega\mu} + \frac{2}{n-1} R_{[\omega\mu]} + \frac{4}{n-1} S_{\omega\mu}^{\beta} S_{\alpha\beta}^{\alpha} \quad . \quad (29)$$

From (29) we see that there exists also an invariant quantity of the second degree, which is in general not zero. If the given connexion is halfsymmetrical

$$S_{\lambda\mu}^{\nu} = S_{[\lambda}^{\nu} A_{\mu]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

then the transformed connexion has the same property and  $T_{\omega\mu\lambda}^{\nu}$  and  $T_{\omega\mu\alpha}^{\alpha}$  take the simple forms

$$T_{\omega\mu\lambda}^{\nu} = R_{\omega\mu\lambda}^{\nu} - \frac{2}{n-1} A_{[\omega}^{\nu} R_{\mu]\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$T_{\omega\mu\alpha}^{\alpha} = V_{\omega\mu} + \frac{2}{n-1} R_{[\omega\mu]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

1) R. K. p. 69.

From (24) follows that  $T_{\omega\mu\lambda}^{\dots\nu}$  is zero when the transformed connexion is euclidean. But vice versa, if  $T_{\omega\mu\lambda}^{\dots\nu}$  vanishes for a halfsymmetrical connexion it is for  $n > 2$  always possible to transform the connexion into an euclidean one by a transformation with an  $A_{\lambda\mu}^{\dots\nu}$  of the form (23).

PROOF.

The supposition  $T_{\omega\mu\lambda}^{\dots\nu} = 0$  is aequivalent to the supposition that  $R_{\omega\mu\lambda}^{\dots\nu}$  has the form

$$R_{\omega\mu\lambda}^{\dots\nu} = \frac{2}{n-2} A_{[\omega}^{\nu} R_{\mu]\lambda} \quad . \quad . \quad . \quad . \quad (33)$$

Now we try to find a field  $q_\lambda$  so that

$$R_{\mu\lambda} = -(n-1) (\nabla_\mu q_\lambda - q_\mu q_\lambda - S_\mu q_\lambda) \quad . \quad . \quad . \quad (34)$$

The conditions of integrability are

$$\left. \begin{aligned} 2\nabla_{[\omega} R_{\mu]\nu} = & -(n-1) R_{\omega\mu\lambda}^{\dots\nu} q_\nu - 2(n-1) S_{[\omega} \nabla_{\mu]} q_\lambda + \\ & + 2(n-1) \nabla_{[\omega} q_{\mu]} q_\lambda + 2(n-1) \nabla_{[\omega} S_{\mu]} q_\lambda \end{aligned} \right\} \quad (35)$$

which equation passes into

$$\left. \begin{aligned} 2\nabla_{[\omega} R_{\mu]\lambda} = & -2(n-1) q_{[\omega} S_{\mu]} q_\lambda - 4(n-1) S_{[\omega} \nabla_{\mu]} q_\lambda + \\ & + 2(n-1) q_\lambda \nabla_{[\omega} q_{\mu]} + 2(n-1) q_\lambda \nabla_{[\omega} S_{\mu]} \end{aligned} \right\} \quad (36)$$

by applying (34) and (35). But by applying the identity of BIANCHI on (33) we get

$$-2 S_{[\omega} R_{\xi\mu\lambda}^{\dots\nu} = \frac{2}{n-1} A_{[\omega}^{\nu} \nabla_{\xi} R_{\mu]\lambda} \quad . \quad . \quad . \quad (37)$$

or, regarding (33)

$$-\frac{4}{n-1} S_{[\omega} A_{\xi}^{\nu} R_{\mu]\lambda} = \frac{2}{n-1} A_{[\omega}^{\nu} \nabla_{\xi} R_{\mu]\lambda} \quad . \quad . \quad . \quad (38)$$

from which follows for  $n \neq 2$

$$2\nabla_{[\omega} R_{\mu]\lambda} = 4 S_{[\omega} R_{\mu]\lambda} \quad . \quad . \quad . \quad (39)$$

or, in consequence of (34)

$$2\nabla_{[\omega} R_{\mu]\lambda} = -4(n-1) S_{[\omega} \nabla_{\mu]} q_\lambda + 4(n-1) S_{[\omega} q_{\mu]} q_\lambda \quad . \quad (40)$$

Now the second identity <sup>1)</sup> for a halfsymmetrical connexion is

$$R_{[\omega\mu\lambda]}^{\dots\nu} = 2\nabla_{[\omega} S_{\mu} A_{\lambda]}^{\nu} \quad . \quad . \quad . \quad (41)$$

<sup>1)</sup> R. K. p. 88.

from which follows, regarding (33)

$$\frac{2}{n-1} A_{[\omega}^{\nu} R_{\mu\lambda]} = 2 A_{[\omega}^{\nu} \nabla_{\mu} A_{\lambda]}^{\nu}, \quad . \quad . \quad . \quad . \quad . \quad (42)$$

for  $n > 2$  equivalent to

$$\nabla_{[\mu} S_{\lambda]} = \frac{1}{n-1} R_{[\mu\lambda]} = -\nabla_{[\mu} q_{\lambda]} + S_{[\mu} q_{\lambda]} \quad . \quad . \quad . \quad (43)$$

In consequence of this equation the conditions of integrability (36) pass into the equation (39) which follows from the identity of BIANCHI, and we have got the following theorem:

*It is for  $n > 2$  then and only then possible to transform a half-symmetrical linear connexion (with  $C_{\lambda\mu}^{\dots\nu} = 0$ ) into an euclidean one by a transformation of the form  $I_{\lambda\mu}^{\nu} = I_{\lambda\mu}^{\nu} + q_{\lambda} A_{\mu}^{\nu}$  if the quantity of curvature has the form  $\frac{2}{n-1} A_{[\omega}^{\nu} R_{\mu\lambda]}$ , in other words if the quantity  $T_{\omega\mu\lambda}^{\dots\nu} = R_{\omega\mu\lambda}^{\dots\nu} - \frac{2}{n-1} A_{[\omega}^{\nu} R_{\mu\lambda]}$  vanishes.*

The quantity  $P_{\omega\mu\lambda}^{\dots\nu}$  (§ 1) is also invariant with transformations characterised by (23).

**Chemistry.** — *Equilibria in systems in which phases, separated by a semi-permeable membrane.* XIX. By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of February 26, 1927).

*Influence of the pressure on osmotic systems.*

Of the many cases, which may occur, if we change the pressure of an osmotic system in which also occur solid phases, we only shall discuss some cases. If we assume that the saturationcurve  $wv$  and the isotonic curves  $wm$  and  $an$  of fig. 1 (communication III) are valid for the pressure  $P$  then we have a.o. the osmotic equilibria:

$$[Y + L_w \mid L_e]_P \quad [Y + L_a \mid L_n]_P \quad \text{fig. 1 III} \quad . \quad . \quad . \quad (1)$$

We now bring the pressure on the right side of the membrane on the higher pressure  $P_1$ . As, like we have seen in the previous communication, then the  $O.W.A.$  of the liquid at the right side of the membrane decreases, we get the osmotic systems:

$$(Y + L_w)_P \xleftarrow{\quad} (L_e)_{P_1} \quad (Y + L_a)_P \xleftarrow{\quad} (L_n)_{P_1} \quad . \quad . \quad . \quad (2)$$

in which the water diffuses in the direction of the arrows. We now assume that a sufficient quantity of  $Y$  is present, so that the system at the left side of the membrane remains. Then the systems (2) pass into the osmotic equilibria:

$$(Y + L_w)_P \mid (L'_e)_{P_1} \quad (Y + L'_a)_P \mid (L'_n)_{P_1} \quad . \quad . \quad . \quad (3)$$

With the isotonic curve of the pressure  $P$  is conjugated an isotonic curve of the pressure  $P_1$ , which must be situated further from the point  $W$  than curve  $wm$ ; we imagine this to be represented by curve  $bo$ . (fig. 1. III). Consequently liquid  $L'_e$  will be represented by the point of intersection  $l$  of the line  $We$  with this curve  $bo$ . The first of the osmotic equilibria (3) becomes, therefore:

$$(Y + L_w)_P \mid (L_l)_{P_1} \quad \text{fig. 1 III}$$

The result of increase of pressure is, therefore, that from liquid  $L_e$  so much water diffuses to the left, till it passes into the liquid  $L_l$ .

The composition of the liquids  $L'_a$  and  $L'_n$  in the second of the systems (3) depends on the ratio of the phases. However, it is sure that  $L'_a$  will be represented by a point of the saturationcurve between  $w$  and  $a$  or by the point  $w$  itself. In the latter case  $L'_n$  is represented by point  $o$ ; the second of the osmotic equilibria (3) becomes then:

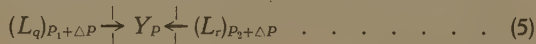
$$(Y + L_w)_P \mid (L_o)_{P_1} \quad \text{fig. 1 III}$$

If, however,  $L'_a$  is represented by a point  $a'$  between  $w$  and  $a$ , then  $L'_n$  will be represented by a point  $n'$  on the side  $WX$  between  $o$  and  $X$ . Those points  $a'$  and  $n'$ , however, are situated not arbitrarily with respect to one another, but the isotonic curve of the pressure  $P$  going through point  $a'$  and the isotonic curve of the pressure  $P_1$  going through point  $n'$  must be conjugated with one another.

We imagine in fig. 1 III two points  $q$  and  $r$  anywhere within the region  $wmXv$ ; as the *O.W.A.* of each of those liquids  $L_q$  and  $L_r$  is larger than that of the solid substance  $Y$  under the same pressure, consequently an osmotic complex



exists, in which no diffusion of water occurs. [For systems with two membranes, compare Comm. X]. We are able now, by bringing the pressure of the left liquid to a definite value  $P_1$  and that of the right liquid to a definite value  $P_2$ , to make the *O.W.A.* of both liquids equal to that of the solid substance  $Y$  under the pressure  $P$ . Then the osmotic complex (4) passes into an osmotic equilibrium, which, if we raise the pressure of both liquids still with  $\Delta P$ , passes into the osmotic system:



in which the water diffuses from left and right towards the solid substance  $Y$ . The double-membrane  $\mid Y \mid$  in system (4) impermeable for water, becomes permeable, therefore, if we raise the pressure of both liquids sufficiently only.

If a sufficient quantity of solid  $Y$  is present, then between the two membranes the system  $Y + L_w$  is formed, and the right and the left liquid lose water as long as their *O.W.A.* becomes equal to that of the liquid  $L_w$  under the pressure  $P$ .

We take under the pressure  $P$  an equilibrium



in which  $F$  is an arbitrary liquid. We now bring the pressure to  $P + dP$ ; keeping the total composition of this system  $E$  constant; then we get the new system:



in which the liquid  $L'$  differs a little in composition from  $L$ ; this difference in composition is completely defined by the change in pressure  $dP$ . With the transition of system  $E$  into  $E'$  the *O.W.A.* changes, as well on account of the change in pressure itself, as on account of the



change of the composition of the liquid; both influence may intensify or debilitate each other. In the previous communication we have seen that this total change  $d\xi$  of the O.W.A. is defined by:

$$d\xi = -\Delta V_E \cdot dP \quad \text{. . . . . (8)}$$

if viz.  $\Delta V_E \cdot \delta w$  is the change in volume, which occurs in the system  $E$  under the constant pressure  $P$ , if we add  $\delta w$  quantities of water. As  $\Delta V_E$ , like we will see further, can be positive, negative and also zero, it follows from (8):

the O.W.A. of an equilibrium  $E = F + L$  of constant composition decreases or increases at increase of pressure, according as  $\Delta V_E$  is positive or negative; if  $\Delta V_E = 0$ , then the O.W.A. does not change with a small increase of pressure.

We will determinate for some cases this change of the O.W.A. Let us firstly take the equilibrium:

$$E = (Y + L_w)_P \quad \text{fig. 1 III} \quad \text{. . . . . (9)}$$

On increase of pressure this passes into the equilibrium:

$$E' = (Y + L'_w)_{P+dP}$$

in which  $L'_w$  is represented by a point  $w'$  which we must imagine in fig. 1 III in the vicinity of point  $w$  on the side  $WY$ . In order to apply formula (8) we have to determinate  $\Delta V_E$  viz. the change in volume which occurs in system  $E$  (consequently under the constant pressure  $P$ ) on addition of water. If we represent the composition of  $L_w$ , which remains unchanged on addition of water to system  $E$ , by  $y$  Mol  $Y + (1-y)$  Mol  $W$ , then, on addition of water the reaction:



occurs. If we represent the molecular volumina of  $L_w$  and the solid substance  $Y$  by  $V$  and  $V_y$ , then we find for  $\Delta V_E$  and also with the aid of (8) for the change of the O.W.A.:

$$\Delta V_E = \frac{V - yV_y}{1 - y} \quad \text{. . (10a)} \quad d\xi = -\frac{V - yV_y}{1 - y} dP \quad \text{. . . (10b)}$$

As we have seen in the previous communication, we can deduce the change of the O.W.A. yet also in another way. The O.W.A. of the liquid  $L_w$  of equilibrium  $E$  is viz. defined by:

$$\xi = -\zeta + y \frac{\partial \zeta}{\partial y} \quad \text{. . . . . (11)}$$

For the O.W.A.:  $\xi + d\xi$  of liquid  $L'_w$  of equilibrium  $E$  is valid then:

$$d\xi = \left( -V + y \frac{\partial V}{\partial y} \right) dP + y t dy \quad \text{. . . . . (12)}$$

in which, however,  $dy$  is defined by  $dP$ . For the equilibrium  $E$  is true viz:

$$\zeta + (1-y) \frac{\partial \zeta}{\partial y} = \zeta_y \quad . \quad . \quad . \quad (13)$$

from which follows for the equilibrium  $E'$ :

$$\left[ V + (1-y) \frac{\partial V}{\partial y} - V_y \right] dP + (1-y) t dy = 0 \quad . \quad . \quad . \quad (14)$$

If we substitute this value of  $dy$  in (13) then we find ( $10^b$ ).

As  $1-y$  is always positive, the sign of  $\Delta V_E$  is defined by the sign of the numerator of ( $10^a$ ). If  $V > V_y$  then the numerator is always positive; if  $V < V_y$  then the sign depends on the value of  $y$ . With values of  $y$ , which do not differ much from 1, the numerator shall be negative then. We may say, therefore, that in general  $\Delta V_E$  shall be positive, unless  $L_w$  should be a concentrated solution, which is formed from its components with contraction of volume.

In order to give another form to  $\Delta V_E$  in ( $10^a$ ) we put:

$$V + (1-y) \frac{\partial V}{\partial y} - V_y = \Delta V_y \quad V - y \frac{\partial V}{\partial y} = \Delta V_w \quad . \quad (15)$$

so that  $\Delta V_y \cdot \delta w$  represents the change in volume of the system  $E$ , when  $\delta w$  quantities of solid  $Y$  dissolve in the liquid;  $\Delta V_w \cdot \delta w$  is the change in volume of the liquid, if this takes in  $\delta w$  quantities of water. We now can write for ( $10^a$ ):

$$\Delta V_E = \Delta V_w + \frac{y}{1-y} \Delta V_y \quad . \quad . \quad . \quad (16)$$

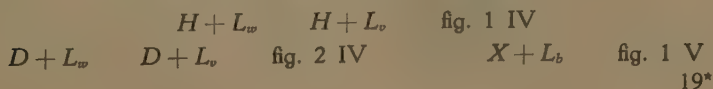
Of the many considerations to which this relation can give rise, we only take the case  $\Delta V_y = 0$ . Instead of ( $10^a$ ) and ( $10^b$ ) we then get

$$\Delta V_E = \Delta V_w \quad . \quad (17^a) \quad d\xi = -\Delta V_w \cdot dP \quad . \quad (17^b)$$

The change of the O.W.A. of system  $E$  on change of the pressure is then the same as if no solid  $Y$  but only the liquid  $L_w$  is present. This also follows from (14); if herein  $\Delta V_y = 0$ , then follows  $dy = 0$ , so that the composition of the liquid  $L_w$  remains constant with a change of pressure  $dP$ ; consequently the quantity of solid  $Y$  remains unchanged also.

As the O.W.A. of the solid substance  $Y$  under the pressure  $P$  is equal to that of the liquid  $L_w$  of system  $E$ , and under the pressure  $P + dP$  equal to that of the liquid  $L'_w$  of the system  $E'$ , ( $10^b$ ), therefore, defines also the change of the O.W.A. on a change of pressure of the solid substance  $Y$ .

Of course similar considerations are valid also for the change of the O.W.A. on a change of pressure of the equilibria:



and consequently also for the change of the *O.W.A.* on a change of pressure of the solid substances *H*, *D* and *X*.

Hence follows, therefore:

the *O.W.A.* of a solid substance *F* can as well increase as decrease on increase of pressure; this depends on the  $\Delta V_E$  of the system  $E = F + L$ , in which *L* is the liquid, which is formed from the solid substance *F* by taking in (losing) water.

We now take under the pressure *P* the equilibrium:

$$E = (Y + L_a)_P \quad \text{fig. 1 III} \quad . . . . . (18)$$

and we bring this, while we are keeping constant the total composition, to the pressure  $P + dP$ ; we then get an equilibrium:

$$E' = (Y + L'_a)_{P+dP} \quad . . . . . (19)$$

in which  $L'_a$  is represented by a point  $a'$ , which we must imagine in fig. 1 III in the vicinity of  $a$  on the line  $Ya$  (within or out of the region  $Ywv$ ). In order to apply formula (8) we have to define  $\Delta V_E$ , viz. the change in volume, which occurs in system (18) on addition of  $\delta w$  quantities of water. However, with this, the composition of the liquid  $L_a$  does not remain constant, as it was the case with liquid  $L_w$  in (9);  $L_a$  viz. changes its composition along curve  $wv$  starting from  $a$  in the direction towards  $w$ .

In order to simplify the calculations, we express the composition of the liquid  $L_a$  in *W*, *Y* and an arbitrary phase *Q*. We write its composition:

$$x \text{ quant. } W + y \text{ quant. } Y + (1 - x - y) \text{ quant. } Q \quad . . (20)$$

so that we have a system of coordinates with *QW* as *X*-axis and *QY* as *Y*-axis. If the equilibrium *E* consists of *n* quantities of  $L_a$  and *m* quantities of *Y*, then we have:

$$\Delta V_E \cdot \delta w = d(nV + mV_y) = V dn + n \frac{\partial V}{\partial x} dx + n \frac{\partial V}{\partial y} dy + V_y dm \quad (21)$$

in which *V* represents the molecular-volume of the liquid  $L_a$ ;  $dx$  and  $dy$  are the changes in concentration, which occur in the liquid  $L_a$ , by the addition of  $\delta w$  quantities of water. As the quantity of water increases with  $\delta w$ , but the quantity of *Y* and *Q* rests unchanged, it follows from this:

$$\delta w = d(nx) = x dn + n dx \quad . . . . . (22)$$

$$0 = d(ny + m) = y dn + n dy + dm \quad . . . . . (23)$$

$$0 = dn(1 - x - y) = dn + dm - \delta w \quad . . . . . (24)$$

in which (24) is simplified with the aid of (22) and (23). As the liquid  $L_a$  is in equilibrium with solid *Y*, the equation:

$$\zeta - x \frac{\partial \zeta}{\partial x} + (1 - y) \frac{\partial \zeta}{\partial y} = \zeta_y \quad . . . . . (25)$$

is valid.

Hence follows:

$$[-rx + (1-y)s]dx + [-xs + (1-y)t]dy = 0. \quad (26)$$

Eliminating  $dn$ ,  $dm$ ,  $dx$  and  $dy$  from the equations (21)–(24) and (26) we find the value of  $\Delta V_E$ . This elimination becomes more simple if we let coincide the arbitrary phase  $Q$  in (20) with the point  $a$  in fig. 1 III; then  $aW$  is the  $X$ -axis and  $aY$  the  $Y$ -axis of the system of coordinates; we now have  $x=0$  and  $y=0$ . Putting those values in (22)–(24) and in (26) and eliminating  $dn$ ,  $dm$ ,  $dx$  and  $dy$  then we find after dividing by  $\delta w$ :

$$\Delta V_E = V + \frac{\partial V}{\partial x} + \frac{s}{t} \left( V_y - V - \frac{\partial V}{\partial y} \right). \quad (27)$$

so that the change of the O.W.A. is known with the aid of (8).

We are able to deduce this more plainly in the following way. We represent again the composition of the liquid  $L_a$  as in (20); its O.W.A. is then defined by

$$\xi = -\zeta - (1-x)\frac{\partial \zeta}{\partial y} + y\frac{\partial \zeta}{\partial y}$$

For an arbitrary liquid, which differs infinitely little from  $L_a$  then is under the pressure  $P + dP$ :

$$d\xi = - \left\{ \begin{aligned} & \left[ V + (1-x)\frac{\partial V}{\partial x} - y\frac{\partial V}{\partial y} \right] dP + \\ & + [-(1-x)r + ys]dx + [-(1-x)s + yt]dy \end{aligned} \right\}. \quad (28)$$

If this new substance under the pressure  $P + dP$  is always in equilibrium with solid  $Y$ , then follows from (25) that  $dx$  and  $dy$  must satisfy an equation, which we get by adding to (26) the term:

$$\left[ V - x\frac{\partial V}{\partial x} + (1-y)\frac{\partial V}{\partial y} - V_y \right] dP. \quad (29)$$

We call this (26<sup>a</sup>). Just as above we now let coincide the arbitrary phase  $Q$  with the point  $a$ . As then  $x$  and  $y$  become zero, (28) and (26<sup>a</sup>) pass into:

$$d\xi = - \left[ V + \frac{\partial V}{\partial x} \right] dP - r dx - s dy. \quad (30)$$

$$0 = - \left[ V + \frac{\partial V}{\partial y} - V_y \right] dP + s dx + t dy. \quad (31)$$

We now must define  $dx$  and  $dy$  in such a way that liquid  $L_a$  of equilibrium (18) passes into  $L'_a$  of (19); as, like we have noted already above  $L'_a$  must be situated on the line  $aY$ , consequently  $dx=0$ . If we substitute this in (30) and (31) and eliminating then  $dy$  we find:

$$d\xi = - \left[ V + \frac{\partial V}{\partial x} + \frac{s}{t} \left( V_y - V - \frac{\partial V}{\partial y} \right) \right] dP. \quad (32)$$

Consequently the same value for  $d\xi$  as follows from (27).

In a similar way we can define the change of the *O.W.A.* in the systems:

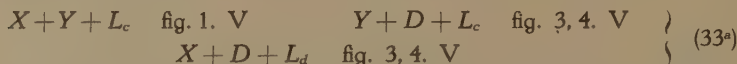


and other systems in which a liquid, saturated with a solid substance.

We now take a liquid, saturated with two solid substances, viz. the equilibrium:

$$E = (F + F_1 + L)_P \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

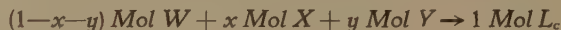
If we bring this, while its composition remains constant, under the pressure  $P + dP$ , then the liquid  $L$  gets a definite change in composition. If we take as simple example for  $E$  a ternary system, f.i. one of the equilibria:



then, on addition of water, the liquid does not change its composition, so that a phases-reaction occurs. If we represent the composition of the liquid  $L_c$  in the first of the equilibria mentioned above, by:

$$x \text{ Mol } X + y \text{ Mol } Y + (1-x-y) \text{ Mol } W \quad . \quad . \quad . \quad (34)$$

then on addition of water the reaction:



occurs. Hence now follows:

$$\Delta V_E = \frac{V - x V_X - y V_Y}{1 - x - y} \quad . \quad . \quad . \quad . \quad (35)$$

so that the change of the *O.W.A.* is defined. Corresponding considerations as for (10<sup>a</sup>) are valid for this value of  $\Delta V_E$ .

If we take the system:



then, on addition of water, also herein a phases-reaction occurs in which, however, the liquid  $L_d$  does not participate. If we represent the composition of the hydrate  $H$  by:  $\beta \text{ Mol } Y + (1-\beta) \text{ Mol } W$ , then this reaction is:

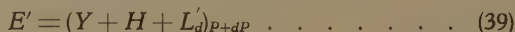


Hence follows:

$$\Delta V_E = \frac{V_H - \beta V_Y}{1 - \beta} \quad d\xi = - \frac{V_H - \beta V_Y}{1 - \beta} \cdot dP \quad . \quad . \quad (38)$$

by which the influence of a change in pressure on the *O.W.A.* of system (36) is defined.

We also can deduce this again in the following way. If we bring the pressure of (36) to  $P + dP$  then arises the equilibrium:





in which  $L'_d$  is represented by a point  $d'$ , which we must imagine in fig. 2  $V$  in the vicinity of point  $d$ . If we represent the composition of the liquid  $L_d$  by (34), then its O.W.A. is defined by:

$$\xi = -\zeta + x \frac{\partial \zeta}{\partial x} + y \frac{\partial \zeta}{\partial y} \quad . \quad . \quad . \quad (40)$$

As the liquid  $L_d$  is saturated with  $Y + H$ , are valid the equations:

$$\zeta - x \frac{\partial \zeta}{\partial x} + (1-y) \frac{\partial \zeta}{\partial y} = \zeta_Y \quad \zeta - x \frac{\partial \zeta}{\partial x} + (\beta - y) \frac{\partial \zeta}{\partial y} = \zeta_H \quad . \quad (41)$$

which we write, with the aid of (40) in the form:

$$\xi = \frac{\partial \zeta}{\partial y} - \zeta_Y \quad \xi = \beta \frac{\partial \zeta}{\partial y} - \zeta_H \quad . \quad . \quad . \quad (42)$$

For the liquid  $L'_d$  of equilibrium  $E'$  under the pressure  $P + dP$  is valid then:

$$d\xi = \left( \frac{\partial V}{\partial y} - V_Y \right) dP + s dx + t dy$$

$$d\xi = \left( \beta \frac{\partial V}{\partial y} - V_H \right) dP + \beta (s dx + t dy)$$

from which follows by elimination of  $s dx + t dy$ :

$$d\xi = - \frac{V_H - \beta V_Y}{1 - \beta} \cdot dP \quad . \quad . \quad . \quad (43)$$

which is in accordance with (38).

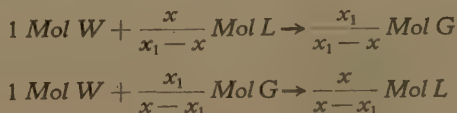
As the systems (33<sup>a</sup>) and (39) define also the O.W.A. of the solid complexes  $X + Y$ ,  $Y + D$ ,  $X + D$  and  $Y + H$ , we also know, therefore, the change of the O.W.A. of those solid complexes, if we bring their pressure from  $P$  to  $P + dP$ .

We now take under the pressure  $P$  an equilibrium:

$$E = (G + L)_P \quad . \quad . \quad . \quad (44)$$

with a vapour  $G$  and a liquid  $L$ . In order to make the calculation not too extensive, we assume that  $E$  is a binary system. We represent the composition of the vapour by  $x$  Mol  $X$  +  $(1-x)$  Mol  $W$  and that of the liquid by  $x_1$  Mol  $X$  +  $(1-x_1)$  Mol  $W$ .

As the phases of this binary equilibrium have a constant composition at constant  $T$  and  $P$ , on addition of water, according as  $x_1 - x$  or  $x - x_1$  is positive, occurs one of the reactions:



If we represent the volume of the vapour by  $V$  and that of the liquid by  $V_1$  then follows:

$$\Delta V_E = \frac{x_1 V - x V_1}{x_1 - x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

by which is defined the change of the O.W.A. of system  $E$  on a change of pressure.

We are able to find this in the following way also.

The O.W.A. of the vapour (and consequently also that of the liquid and that of the total system  $E$ ) is defined by:

$$\xi = -\zeta + x \frac{\partial \zeta}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

If we bring this system under the pressure  $P + dP$ , while its total composition remains constant, then we get the equilibrium:

$$E' = (L' + G')_{P+dP} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

of which the O.W.A. is defined by

$$d\xi = \left( -V + x \frac{\partial V}{\partial x} \right) dP + x r dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

and in which  $dx$  has a value defined by  $dP$ . For the equilibrium  $E$  is true viz.:

$$\zeta - x \frac{\partial \zeta}{\partial x} = \zeta_1 - x_1 \frac{\partial \zeta_1}{\partial x_1} \quad \frac{\partial \zeta}{\partial x} = \frac{\partial \zeta_1}{\partial x_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

from which follow for equilibrium  $E'$  the equations:

$$\begin{aligned} \left( V - x \frac{\partial V}{\partial x} \right) dP - x r dx &= \left( V_1 - x_1 \frac{\partial V_1}{\partial x_1} \right) dP - x_1 r_1 dx_1 \\ \frac{\partial V}{\partial x} \cdot dP + r dx &= \frac{\partial V_1}{\partial x_1} dP + r_1 dx_1 \end{aligned}$$

If we substitute, after having eliminated  $dx_1$ , in (48)  $dx$  then we find:

$$d\xi = \frac{x_1 V - x V_1}{x - x_1} \cdot dP \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

which is corresponding with the value of  $d\xi$  which follows from (45).

In order to define the sign of the coefficient of  $dP$  we distinguish two cases.

1. The vapour  $G$  has a smaller amount of water than the liquid  $L$ ; consequently we have  $1 - x < 1 - x_1$  or  $x > x_1$ . As in general the volume  $V$  of the vapour is many hundred times as large as the volume  $V_1$  of the liquid,  $x_1 V - x V_1$  will be, therefore, generally positive, unless in the special case that  $x$  (viz. the concentration of the substance  $X$  in the vapour) is many hundred times as large as  $x_1$  (the concentration of the substance  $X$  in the liquid).

If we exclude this case, then the coefficient of  $dP$  in (50) is positive, therefore.

2. The liquid  $L$  has a smaller amount of water than the vapour  $G$ ; consequently we have  $1 - x_1 < 1 - x$  or  $x_1 > x$ . The numerator of (50) now always is positive, the denominator negative, so that the coefficient of  $dP$  is negative.

Consequently we can say:

the O.W.A. of the binary system  $G + L$  becomes larger on increase of pressure, when the vapour has the smallest amount of water, and smaller if this is the case with the liquid.

We now take the osmotic equilibrium:

$$(Y + L)_P \mid (Y + L)_P \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

in which on both sides of the membrane the system (9) discussed above, under the pressure  $P$ . We distinguish two cases.

1. The solubility of the solid substance  $Y$  increases on increase of pressure.

If we assume that the total concentration of the substance  $Y$  is smaller at the left side of the membrane than on the right side, then, as on increase of pressure the quantity of solid  $Y$  decreases on both sides of the membrane, under a definite pressure  $P_1$  ( $P_1 > P$ ) the solid  $Y$  at the left of the membrane will disappear, while it is still present at the right side; then (51) passes into the osmotic equilibrium:

$$(L_1)_{P_1} \mid (Y + L_1)_{P_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

in which the liquid has the same composition on both sides. If we increase the pressure still with  $\Delta P$ , then the liquid at the left side of the membrane remains unchanged; at the right side of the membrane, however, it gets a somewhat other composition  $L'_1$ . The O.W.A. at the left and at the right side of the membrane then increases with:

$$(d\xi)_l = -\Delta V_W \cdot dP \quad (d\xi)_r = -\Delta V_E \cdot dP \quad . \quad . \quad . \quad (53)$$

in which  $\Delta V_W$  and  $\Delta V_E$  are defined by (10<sup>a</sup>), (15) and (16). It now follows from (53):

$$(d\xi)_l - (d\xi)_r = \frac{y}{1-y} \cdot \Delta V_g \cdot dP \quad . \quad . \quad . \quad . \quad . \quad (54)$$

As the solubility of the substance  $Y$  increases on increase of pressure,  $\Delta V_Y$  must be negative, as is apparent from (14) and (15); consequently follows from (54):

$$(d\xi)_l < (d\xi)_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

On an increase of pressure  $\Delta P$  ( $\Delta P > 0$ ) consequently (52) passes into the osmotic system:

$$(L_1)_{P_1+\Delta P} \mid (Y + L'_1)_{P_1+\Delta P} \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$



**Zoology.** — *On the Larval Forms of Rhizocephala.* By H. BOSCHMA.  
(Communicated by Prof. C. PH. SLUITER.)

(Communicated at the meeting of February 26, 1927).

From a taxonomic point of view the larvae of the Rhizocephala are of special interest, as their structure gives sufficient evidence for the relationship of this group of parasitic animals with the Cirripedia. Like the nauplii of *Lepas* and *Balanus* the corresponding larval stages of *Sacculina* and *Peltogaster* possess well developed frontal horns, a peculiarity which is not found in other groups of Crustacea.

In all Rhizocephala the eggs after being hatched remain for a considerable time in the mantle cavity and here the first stages of development take place. The structure and the development of the larval stages of *Sacculina carcini* are well known (DELAGE, 1884). In this species the larvae leave the mantle cavity at the first nauplius stage. Between this stage and the cypris there are three more nauplius stages, each of which has a more or less different shape. Especially between the first and the second nauplius stages there is a considerable difference in structure (cf. DELAGE, 1884, Pl. XXII, figs. 1 and 2).

In general the specific characteristics of the adult Rhizocephala are not easily defined. Often the internal structure, especially the form and situation of the testes and the colleteric glands may give sufficient data for the description, but in other cases, as in the Sacculinidae, there are a great number of species which correspond completely in their internal structure. Now in this family many species are sufficiently characterized by the appendages of the external cuticle, which have a constant specific form, or are at least subject to very little variation in different specimens belonging to the same species. One would expect now that the larvae of different species of Sacculinidae would possess distinct specific characteristics also and even more conspicuously than the adult parasites.

Among the material of Rhizocephala from the collections of a number of zoological museums, which I have at present at my disposal, I have found the larvae of seven species of *Sacculina* in the mantle cavity of these parasites. They are therefore at the first nauplius stage, as also results from their structure. All these larvae, some of which still are in an excellent state of preservation, in general have the same form. The nauplii of three species are represented in fig. 1. One of these (fig. 1b) is the larva of *Sacculina carcini* (from a specimen attached to *Pachygrapsus marmoratus* from the Mediterranean Sea). The other, smaller specimen (fig. 1a) is from a small still undescribed species of *Sacculina* from Japan, the third (fig. 1c) is the nauplius of *Sacculina exarcuata* Kossm. from



Hong Kong. The general form of these three nauplii is very similar: they all have well developed frontal horns and small triangular ventral



Fig. 1. Larvae of different Rhizocephala.

a—c, different species of *Sacculina*; d, *Peltogaster*; e, *Clistosaccus*; f, *Sylon*.  $\times 130$ .  
In a—d the appendages of one half are omitted. For further particulars see text.

appendages at the extremity of the abdomen. In two of them (b, c) a distinct eye spot is to be seen; in the third species (a) this organ is completely wanting, which probably is a characteristic of the species. Another peculiarity, which, however, is subject to rather extensive individual variation, is the difference in size between the larvae of a certain species and those of another. The larvae of the Japanese species (fig. 1a) are much smaller than those of *Sacculina carcini* (fig. 1b), whilst the latter again are rather small in comparison with those of *Sacculina exarcuata* (fig. 1e). I did not find specific differences in the structure of the three paired appendages of these and other nauplii of *Sacculina*. But also in other respects the first nauplius stage in this genus gives very few data for a distinction of different species by characteristics found in the larvae. They may possess some peculiarities which certainly are of specific value, e.g., the presence or absence of an eye spot, but in general their differences are too inconspicuous to be of use for taxonomic questions. As to the differences in size found between corresponding larval stages of different species we cannot maintain that these are of any specific value. The eggs of large specimens of a certain species of *Sacculina* may be somewhat larger than those of small specimens belonging to the same species, and the resulting larvae in both cases may be of somewhat different size.

Probably the second and later nauplius stages of different species of *Sacculina* will show more striking differences. It is quite possible that their structure may give valuable data for the taxonomy. Unfortunately, however, the later nauplius stages of two species only are known, viz., those of *S. carcini* and of *S. neglecta*. The free swimming nauplius of the latter has been described by SMITH, and from this author's figure (SMITH, 1906, Pl. 4, fig. 17) results that really this larva differs from the corresponding stage of *S. carcini*. Especially the shape of the ventral abdominal appendages is different in both forms: in *S. neglecta* they are rather broad and are provided with dentations at the internal surface only, whilst in *S. carcini* they have a more slender shape and have lateral hairs at the external as well as at the internal surface.

In the Peltogastridae the larval stages may prove to be of very great interest for taxonomic purposes, e.g., in the case of *Peltogaster curvatus* Kossm. and *Peltogaster paguri* Rathke. The structure and anatomy of the adult forms of both of these species are very similar, though usually the external shape is somewhat different. The only characteristic of specific value in the adult animals is the presence of small papillae at the extremities of the mantle in *P. curvatus* (DUBOSCQ, 1912), which as a rule do not occur in *P. paguri*. There are, however, some remarks in the literature on *P. paguri* which point to the fact that also in the latter species sometimes these papillae may develop (cf. LILLJEBORG, 1859, GUÉRIN—GANIVET, 1911). But in the two species under consideration the larvae furnish a characteristic which in my opinion decides at once that we have to regard the above mentioned forms as different species. SMITH (1906) gives a short description and a figure of the free swimming nauplius of *Peltogaster curvatus*, which is characterized by much elongated frontal horns. These horns are directed forward (cf. SMITH, 1906, Pl. 4, fig. 18). SMITH used this peculiarity even as a characteristic in the diagnosis of the genus *Peltogaster*. Here, however, this author was not right, for in other species of this genus the frontal horns are much smaller and they are directed laterally or backward. The best known larvae are those of *P. paguri* and *P. sulcatus*, especially by the investigations of NILSSON—CANTELL (1921). On account of the different size and shape of the frontal horns these larvae may be at once distinguished from those of *P. curvatus* and this fact gives sufficient evidence for the view that *P. paguri* and *P. curvatus* are distinct species.

The first nauplius stage of different species of *Peltogaster* again often has a very similar form. Fig. 1d represents such a larva of *Peltogaster gracilis* Kr. from Chili, which in very subordinate peculiarities only differs from that of *P. sulcatus* (cf. NILSSON—CANTELL, 1921, fig. 14b). In the larva of *P. gracilis* also the frontal horns are small and have a lateral direction. The swimming hairs are somewhat longer than those of *P. sulcatus*.

As in *Sacculina* and in *Peltogaster* in most of the other Rhizocephala

the larva develops in the mantle cavity till the first nauplius stage. This is known from *Peltogasterella* (KRÜGER, 1914), *Lernaeodiscus* (MÜLLER, 1862), and *Parthenopea* (KOSSMANN, 1873). On the other hand in the genus *Thompsonia*, which in many other respects also differs from the other genera of Rhizocephala, the nauplius stage is omitted and the larva develops directly into the cypris stage (HÄFELE, 1912; POTTS, 1915). Besides this genus there is still another in which the eggs develop directly into cypris larvae, viz. *Sesarmaxenos* (ANNANDALE, 1911), a rather imperfectly known parasite from a fresh water crab found at the Adaman Islands.

Till now the larval stages of the species belonging to the genera *Sylon* and *Clistosaccus* (the name *Apeltes* is a synonym of the latter genus) were completely unknown. SMITH (1906) as well as HOEK (1909), the authors of the two most extensive publications on the Rhizocephala as a whole, express the opinion that there are in the literature no data concerning the larvae of these genera. In the material of the Copenhagen Zoological Museum, which I am studying for systematic purpose, I have found the larvae of both of the above mentioned genera. In *Clistosaccus* as well as in *Sylon* the development of the eggs results in the formation of the cypris larvae, here also a nauplius stage is omitted. The larva of *Clistosaccus paguri* (fig. 1e) is very similar to that of *Sesarmaxenos*; it has six pairs of pleopods with strong swimming hairs and a pair of well developed antennae. Though the larvae in my material are already fully developed many of them still are contained in the egg membrane, which proves that they have not developed from nauplii. Concerning the larvae of *Sylon hippolytes* (fig. 1f, the figured larva was taken from the mantle cavity of a parasite on *Spirontocaris lilljeborgi*) I want to state that the specimens of my material in all probability are not yet fully developed cypris larvae. Already a distinct pair of antennae and the six pleopods of each side are to be seen, but probably the larva of *Sylon* has to undergo some more changes before leaving the egg membrane. The swimming hairs of the pleopods are here also well developed.

Knowing now the chief points of the structure of the larvae of *Clistosaccus* and *Sylon* one can understand a few remarks published by KRÖYER (1855). This author has seen the larvae of three different species of Rhizocephala, which he could compare with the figure of the larva of *Sacculina* published by CAVOLINI (1787). KRÖYER examined the larvae of *Peltogaster gracilis*, of *Sylon hippolytes*, and of a species which he called "another species of *Peltogaster*". Now in former times the species *Clistosaccus paguri* has often been confounded with *Peltogaster paguri*, with which it often bears a strong external likeness. From another paper by KRÖYER (1842) we may deduce that he certainly has possessed specimens of the arctic form *Clistosaccus*. According to KRÖYER (1855) the larva of *Peltogaster gracilis* is very similar to that of *Sacculina*. This undoubtedly is true and is proved by a comparison of the figs. 1b and d in the present

paper. The larvae of *Sylon* and of "the other species of *Peltogaster*" were characterized by KRÖYER as forms which probably represented a later stage in the development. The larva of *Sylon* was very similar to that of "the other species of *Peltogaster*", but both differed strongly from those of *P. gracilis* and *Sacculina*. We may therefore safely conclude that KRÖYER as early as 1855 already knew the larvae of *Sylon* and *Clistosaccus* ("the other species of *Peltogaster*"), but owing to his short and incomplete remarks on these larvae they remained unknown in literature.

It is an interesting fact that protection of the brood, which occurs in so many arctic forms of animals, manifests itself also in the Rhizocephala: *Clistosaccus paguri* and *Sylon hippolytes* are true arctic species, whilst the Sacculinidae and the species of the genus *Peltogaster* do not occur in the far north. *Sesarmaxenos* is a tropical species, but it lives in fresh water and in this medium the development of the larvae also in other groups of animals is shorter than that in marine animals of the same group. The species of the genus *Thompsonia* are strongly specialized forms and differ structurally in many details from the other Rhizocephala. It is difficult to explain why also in this genus the eggs develop directly into a cyprid stage.

#### LITERATURE.

N. ANNANDALE, 1911. Note on a Rhizocephalous Crustacean from Fresh Water and on some specimens of the Order from Indian Seas. Rec. Ind. Mus., Vol. 6.

F. CAVOLINI, 1787. Sulla Generazione dei Pesci e dei Granchi. Napoli. (This work was not accessible to me).

Y. DELAGE, 1884. Évolution de la Sacculine (*Sacculina carcini* Thoms.). Arch. Zool. Exp. Gén. (2), T. 2.

O. DUBOSCQ, 1912. Sur les Peltogastrides des Côtes de France. Arch. Zool. Exp. Gén. (5), T. 9, Notes et Revue.

J. GUÉRIN—GANIVET, 1911. Contribution à l'Étude systématique et biologique des Rhizocéphales. Trav. Sc. du Lab. de Zool. et de Phys. Marit. de Concarneau, T. 3.

F. HÄFELE, 1912. Anatomie und Entwicklung eines neuen Rhizocephalen *Thompsonia japonica*. Abh. II. Kl. K. Ak. Wiss. München, Suppl. Bd. 2.

P. P. C. HOEK, 1909. Die Cirripeden des nordischen Planktons. Nordisches Plankton, 8.

R. KOSSMANN, 1873. Suctoria und Lepadidae. Habilitationsschrift, Würzburg. (also in: Arb. zool.-zoot. Inst. Würzburg, Bd. 1, 1874).

H. KRÖYER, 1842. Monografisk Fremstilling af Slaegten *Hippolyte's* nordiske Arter. K. Dansk. Vidensk. Selsk. Naturvid. og math. Afh., Dl. 9.

H. KRÖYER, 1855. Nogle Bemaerkninger om en højst ufulstaendigt bekjaendt Kraebdsdyr-gruppe. Overs. K. Dansk. Vidensk. Selsk. Forh. i. Aaret 1855.

P. KRÜGER, 1914. Ueber ostasiatische Rhizocephalen. Abh. II. Kl. K. Ak. Wiss. München, Suppl. Bd. 2.

W. LILLJEBORG, 1859. Les Genres *Liriope* et *Peltogaster*, H. Rathke. Nova Acta Reg. Soc. Scient. Upsal. (3), Vol. 3.

F. MÜLLER, 1862. Die Rhizocephalen, eine neue Gruppe schmarotzender Kruster. Arch. Naturg., 28. Jahrg., Bd. 1.

C. A. NILSSON—CANTELL, 1921. Cirripedenstudien. Zur Kenntnis der Biologie, Anatomie und Systematik dieser Gruppe. Zool. Bidr. f. Uppsala, Bd. 7.

F. A. POTTS, 1915. On the Rhizocephalan Genus *Thompsonia* and its Relation to the Evolution of the Group. Papers Dep. Mar. Biol. Carnegie Inst. Washington, Vol. 8.

G. SMITH, 1906. Rhizocephala. Fauna und Flora des Golfes von Neapel, 29. Monographie.



**Anatomy.** — *On the occurrence of Vascularized Enamel-organs.* By  
J. MULLER. (Communicated by Prof. L. BOLK.)

(Communicated at the meeting of December 18, 1926).

Is the enamel-organ vascularized, or is it not? Up to this day opinions differ concerning this seemingly simple question.

In former years it was generally assumed that an enamel-organ was never vascular i.e. provided with bloodvessels. However, in later years some cases were described in which vascularization was beyond dispute. The present paper discusses a new case which Prof. BOLK submitted to me for examination. I shall preface the description of the case with a review of what has already been published on this vascularization.

When looking up the literature of the vascularization of the enamel-organ, it strikes us that this problem has but seldom received general notice, although it is of great importance for the nutrition of the ameloblasts. Most times that vascularization was treated of only incidentally; a systematic study has never been made of the occurrence of bloodvessels in the enamel-organ.

Indeed, it was generally known that in the later stages of the development, there is a closer relation between the bloodvessels and the stratum intermedium. When in the course of development the reduction of the enamel-pulp begins, the external epithelium leans up against the cells of the stratum intermedium, so that the bloodvessels are separated from the intermediate cell-layer only by this external epithelium. Nay, some researchers even hold that the external epithelium disappears altogether, and the capillaries come into direct contact with the stratum intermedium.

LEON WILLIAMS even describes, in this developmental stage of the *rat*, bloodvessels within the stratum intermedium. MUMMERY says of WILLIAMS: "he describes bloodvessels in the stratum intermedium at this stage (the later stages of enamel formation, when the stellate reticulum has disappeared), and that there are bloodvessels in this layer is fully evidenced by the photographs in illustration of his paper".

However, my object in this paper is to discuss the occurrence of vascularization in still fully developed enamel-organs, i.e. in stages in which the internal and the external epithelium are still separated by a well-developed pulp, and the latter does not show any sign of reduction. An instance of this was not described before 1889. Before entering upon my subject I will bring forward some reports regarding vascularization, that appeared before that date.

In the edition of 1870 WEDL wrote in his *Manual* in accordance with the then current opinion: "Das Schmelzorgan ist gefässlos."



In complete harmony with this pronouncement is LEGROS and MAGITOT's ('79) description of the enamelorgan in a fetus of a *horse* in the 3<sup>d</sup> month of the development and in a newborn *dog*, in which they say: "cet organe étant absolument dépourvu aussi bien de système vasculaire que de système nerveux."

Just as the above researchers KLEIN and NOBLE SMITH ('80) detected that in a *dog* "no vessels are present in this middle membrane".

The first positive inquiry regarding a vascularized enamel-organ was made by POULTON ('89). This author studied the development of the dentition in *Ornithorhynchus paradoxus* and found here vascularization of the enamel-organ, as is borne out by the following quotation:

"I could not detect capillaries in the stratum intermedium — although they are certainly present in the stellate reticulum and are sometimes seen very near this layer (the stratum intermedium)", and a little further he says: "It is quite certain that blood-vessels are present in this layer (i.e. the enamel-pulp) and they extend into all parts of it." He could trace out the vessels from the adjacent connective tissue through the external epithelium down to the enamel-pulp, and observed that coincidentally connective tissue forced its way in. The capillaries looked like strands of spindle-shaped cells with a narrow lumen, often difficult to distinguish.

The author further remarks that HOWES has found bloodvessels in the enamel organ of the *rat* and showed him the preparations.

Strange to say, HILL and WILSON who, in connection with POULTON's study, examined more closely the dental development in *Ornithorhynchus*, do not make mention of a vascularization in their extensive description.

The next-following observation, recorded in the literature is of much later date (1911). In this long interval we only read, in a description given by ADDISON and APPLETON of a case of vascularization found by the authors themselves, that PAUL ('96) has observed the occurrence of vascularized enamel-organs in the *calf* and the *lamb*, but that he could not find in either bloodvessels in the enamel pulpa.

As stated just now, only in 1911 do we meet with the description of a second instance, and while POULTON found bloodvessels in the enamel organs of rudimentary teeth, such as are met with in *Ornithorhynchus*, HOPEWELL-SMITH and MARETT-TIMS now discovered bloodvessels in the enamel organ of teeth in a later developmental stage. They examined the development of the dentition in *Macropus billardieri*. According to their description the bloodvessels penetrate in several places through the external epithelium, and the section of these vessels was large enough to show blood-corpuscles. They did not find any bloodvessels in the stratum intermedium. The authors correlate this vascularization with the peculiar feature in the dental development of *Macropus*, that the ameloblasts begin to form enamel even prior to the formation of dentin. SMITH and TIMS write: "It is extremely probable that the unusual vascularity of the enamel-organs is correlated with the precocious development of the enamel."

A similar observation has been made by MUMMERY in an allied species, viz. in *Macropus rufus*. In his "The microscopical and general anatomy of the teeth" he says on this point: "bloodvessels are seen crossing the external epithelium and also lying within the stellate reticulum, an appearance seen in many sections."

An extensive description of a vascularized enamel-organ in another marsupial — viz. *Phascolarctos cinereus* — is given by BOLK ('15). He lays stress on the fact that a network of bloodvessels forms in the surrounding connective tissue at the transition from the enamel-septum to the external epithelium. From this network vascula force themselves into the enamel septum and thence into the pulpa, but also in other places bloodvessels grow through the external epithelium. Two layers are distinctly noticeable at the vascular reticulum in the pulpa the one more or less superficial, consisting of the larger vessels, and the other, situated deeper down, contiguous to the ameloblasts, and consisting of narrower vessels. The author adds that between the enamel pulp and the amelogenetic cells there lies only a thin layer of less differentiated cells, which should not be considered as a true stratum intermedium.

Also in the premolars of the *calf* vascularized enamel-organs were found by BROOMELL and FISCHER ('17), as is briefly indicated in a communication by ADDISON and APPLETON. A peculiar feature of this case is that the vessels could be traced as far as the stratum intermedium.

The observations of HOPEWELL-SMITH, MARETT TIMS and MUMMERY on the vascularization of the enamel-organs in the genus *Macropus* have been corroborated by CARTER. He says: "HOPEWELL-SMITH and MARETT TIMS referring to the vascularity of the enamel-organ in *Macropus billardieri*, state that they were unable to trace the capillaries beyond a point midway between the outer and inner enamel epithelium, but in my preparation vessels of a lumen from  $20\ \mu$  to  $30\ \mu$  in diameter are frequently seen lying in contact with the cells of the stratum intermedium."

CARTER found these relations in *Macropus ruficollis*.

ED. PASCALIS ('18) and SKILLEN ('21) worked with a technique quite new for this inquiry. They injected their preparations. PASCALIS studied the vascularization of the teeth during the development, and to this end he examined a *cat's*-embryo of 10 cm in length — a newborn *cat* — one of 1 week, one of 2, 3, 4, 5 weeks and a full-grown *cat*; also a newborn *dog* and a *human fetus*.

With all of them he finds: "les vaisseaux adamantins forment, à la surface de l'organe de l'émail un réseau comparable à celui de la pulpe, mais à mailles plus serrées et formées de capillaires de plus gros calibre."

SKILLEN also worked with much material and examined the development of the dentition in the *pig* — the *dog* — the *sheep*, the *rabbit*, the *cavia* and some earlier stages in *man*. He used injected and non-injected preparations, but could not detect any bloodvessels in the enamel-organ.

JORDAN's researches date from the same year. He studied an "anlage" of

a tooth from the lower jaw of a *cat* and felt convinced that vascularization of the enamel-organ does not occur here.

In a second publication he makes mention of his observations of an incisor of the *white rat* in a period from 1 to 7 days after birth and of the tooth-germ from the lower jaw of a newborn *cat*, and of *cats*, 1—2 and 3 weeks old. He finds that the incisor of the *white rat* is not furnished with bloodvessels, while in the *cat* the bloodvessels push the outer epithelium slightly forward. Hereby vascular papillae are generated, which were first described by WILLIAMS, and are called after him WILLIAMS's papillary layer.

In 1922 ADDISON and APPLETON record the occurrence of vascularization of the enamel-organ of the first molar in the upper-, and the lower-jaw. Their material comprised different stages in the development of the *white rat*. On the 21<sup>st</sup> day of the intrauterine life vessels have really penetrated into the pulpa. In the following stages the vessels have grown deeper into the pulpa, and approach the stratum intermedium. In the three-day-old *rat* the vessels are contiguous to the stratum intermedium. It is interesting that ..... "at the time of the entrance of the first bloodvessels into the enamel-organ (21 days' fetus) no enamel has yet been formed. As the number of entering bloodvessels increase, and with their deeper penetration into the stellate reticulum, amelogenesis begins. Not until the bloodvessels have penetrated to the vicinity of the stratum intermedium is there a well-defined layer of enamel to be seen."

The occurrence of perivascular lymph-spaces, first described by BOLK ('15) could be confirmed by them.

Although many of the above descriptions are conclusive for the occurrence of vascularized enamel-organs, still it is discredited by some observers.

JORDAN says that he is very sceptical as to the cases of vascularization described. He does not consider them conclusive.

PRENANT ('24) sides with JORDAN and holds that the would-be capillaries apparently found in the pulpa are nothing else but severed vascular papillae of WILLIAMS, so that the vessels are still enclosed by outer epithelium.

I do not agree with these authors, and feel convinced that bloodvessels may indeed occur in the enamel pulp, but that this depends on the species and the age of the animal. Vascularized enamel-organs are by no means present in every animal species, and they are found only in a definite developmental stage.

When summarizing the various publications on the vascularization of the enamel-organ it appears, that most authors deny its occurrence, but that the experience of some workers is that an entrance of bloodvessels into the enamel-organ can unmistakably be witnessed. What strikes us most is that the same positive results were obtained with various species of Marsupialia, viz. with *Phascolarctos cinereus*, *Macropus rufus*, *Macropus*

ruficollis and *Macropus billardieri*, in all of which bloodvessels were found in the enamel-pulpa. This indicates that with the species of Marsupialia the occurrence of this vascularization may reasonably be expected to be much more general than has hitherto been found.

Secondly the conviction grows upon us, that there is some connection between this supply of bloodvessels and amelogenesis.

HOPEWELL-SMITH and MARETT TIMS already pointed to this, as they found that the vascularization was attended with a precocious development of the enamel. ADDISON and APPLETON also came to the conclusion that: "The vascularization of the enamel-organ is closely connected chronologically with the beginning of amelogenesis.

Let us state in the third place that BOLK established the absence of a stratum intermedium in *Phascolarctos cinereus*, and correlated this with the precocious vascularization.

CARTER arrives at the same conclusion in his examination of *Macropus ruficollis*, when he says: "when growth is very rapid the stellate reticulum may be seen in contact with the ameloblasts without the intervention of the stratum intermedium."

That this is not always the case, is shown by the inquiry of ADDISON and APPLETON, in which the stratum intermedium was present in all the stages they studied.

In the fourth place we may ask why the vascular system comes so soon into contact with the stratum intermedium and the ameloblasts.

When there is no vascularized enamel-organ, we see that, after the reduction of the pulpa, the bloodvessels come into close contact with the ameloblasts. The pulpa provided nourishment for the ameloblasts, and after the reduction of the pulpa this function is assumed by the bloodvessels. What, however, should we think of those cases in which the enamel-organ is still fully developed and the bloodvessels penetrate into the pulpa and are contiguous to the stratum intermedium? At the conclusion of this paper I hope to recur to this point.

My own observation of vascularization of the enamel-organ was made on an embryo of *Dactilomys*.

Frontal sections of 20  $\mu$  of the preparation were stained with hematoxylin. Three tooth-germs were present in the upper- and the lower-jaw, on the left as well as on the right.

*Dactilomys* belongs to the Hystricomorpha, a subdivision of the Rodentia, whose complete dental formula is

$$I \frac{1}{1} P \frac{4}{4} M \frac{1 \frac{2}{2} 3}{1 \frac{2}{2} 3}.$$

There is no succession, so that the germs of I—P and  $M_1$  are present.

Amelogenesis has not yet begun, but dentinification has. In the incisors the dentin already consists of a thick layer, as may be seen in fig. 1 representing a section through the "anlage" of an incisor from the



upper jaw. The enamel-organ has slightly receded from the adjacent connective tissue, also a fissure has arisen on either side of the dentin through the retraction of the dental papilla and the ameloblasts. A stratum intermedium is well-developed. The section has cut the tooth-band only a little near the mouth-epithelium.

However, what attracts our attention most is the occurrence of blood-vessels in the enamel-pulp. Vascularization can be seen in all the sections of this tooth. The bloodvessels may distinctly be traced from the adjacent connective tissue into the pulp. The vascula that appear as strands of

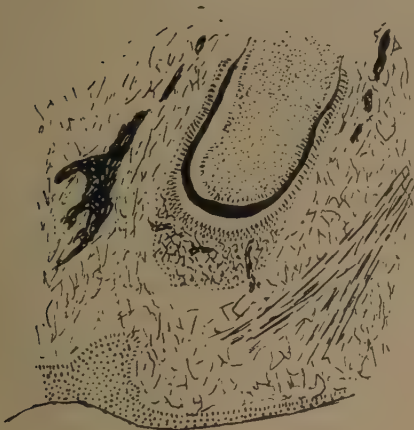


Fig. 1.



Fig. 2.

spindle-shaped cells, pass through the enamel-organ in all directions, but never enter the stratum intermedium. They can only be observed as far as this layer.

When proceeding to the following sections we notice (Fig. 2) another peculiarity of the vascularized enamel-organ, viz. that here vessels penetrate into the enamel-organ exclusively on the buccal side. This, however, is the exception, but when comparing the various sections of the incisivi, we shall observe that most vessels make their way into the enamel organ on the buccal side of the dental ledge. This is also the case with the premolars.

Something like this was also observed by ADDISON and APPLETON in their preparations. They ascribed it to the structure and the position of the enamel-organ. They correlated this phenomenon in the first place with the difference in the length of the lingual and the buccal sides of the organ, measured from the point of attachment of the dental ridge to this organ. The authors say: "This difference in length of the lingual and buccal portions, seems to be of importance in connection with the site of entrance



of the blood-vessels into the enamel-organ." This difference in length also occurs in my preparations.

But the relations in my case are the reverse of those found by ADDISON and APPLETON. This may be seen from a comparison of fig. 2 with what they write. In the figure the buccal area is considerably longer than the lingual area. The said authors write: "The fact that the palatal surface of the external epithelium is considerably wider than the buccal or lateral surface would (other things being equal) in itself afford opportunity for the entrance of more bloodvessels through the palatal surface, than through the buccal surface."

The authors have preceded these descriptions by the statement that the dental ledge is attached medially to the enamel-organ and say that: "the lingual surface of the dental ledge is almost in a straight line with the adjoining lingual surface of the external enamel epithelium", so that I may be allowed to conclude that this reversion of the relations is the consequence of a misconception, and that, therefore, the relations in their preparations agree with those I found in *Dactilomys*.

Secondly ADDISON and APPLETON ascribed it to the position of the dental germ relative to the alveolus of the tooth, as is expressed in the following quotation: "This advantage is further increased by the fact that the buccal surface of the external enamel epithelium is for almost its full extent in close contact with the osteogenetic membrane or with the developing bone itself of the maxilla. This proximity would seem to interfere with the free access of blood-vessels to this side of the enamel-organ."

After applying the correction alluded to above we see, then, that the relations are the same also here.

On close inspection of fig. 2 our attention is arrested by the presence of a rudimentary tooth. Buccally to the dental ledge we see it as a conoid mass of dentine. It is contiguous to the epithelium of the mouth-cavity and I could not detect an enamel-organ, but I observed that the connective tissue has thickened round the toothlet.

Similar rudimentary toothlets were present in the upper- and in the lower-jaw. They were always disposed labially to the germ of the incisors. They have often been described in papers on rodents and have been considered as the remains of a milk dentition.

Fig. 2 shows us still more. Hitherto we have always been speaking of "dental ledge". May we do so? In the figure the ledge is obviously many cells broad and it seems to me that the enamel-organ is rather suspended directly to the mouth-epithelium, without the intervention of a dental ledge. The epithelial cells are seen to pass gradually into the pulpal cells.

The germ of the incisivi from the lower-jaw is much smaller than of those of the upper-jaw. Fig. 3, representing a cross-section through such an incisor, shows us a veritable network of vessels in the enamel-pulp. This

vascular reticulum has penetrated into all the parts of the pulpa, and is supplied by vessels that have grown through the outer epithelium in several

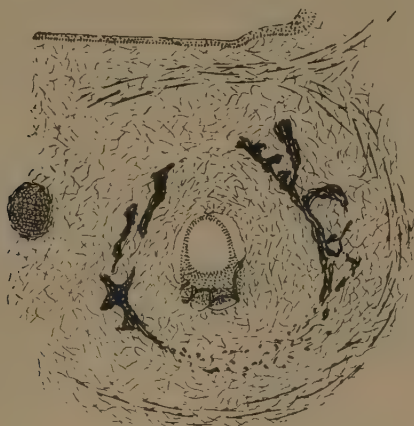


Fig. 3.

places. The stratum intermedium is very broad, the vessels do not enter here either. What we do see again, is that the vessels are seen to run up against this layer. The outer-, and the inner-epithelium are separated only on one side from the enamel-organ by pulpal tissue. On the other side they are adjoined. This is easy to understand, if we reflect that we have to do here with the rudiment of an incisor of a rodent, which displays amelogenesis only on the labial surface.

The section has not cut the dentine. Owing to the retraction it does not lie in the plane of the figure.

I fancied I had also found in this preparation the perivascular lymphspaces described by Bolk. We see them already in fig. 2, but better still in fig. 4. The vessels are surrounded here by broad spaces. Are these the same lumina that Bolk observed? The spaces shown by the figure may be artefacts. The enamel-pulp is extremely aqueous. Fixation of course makes it shrivel, thus causing the spaces. This view is still favoured by the fact that they are connected with that fissure round the dental germ, which is no doubt the result of the shrivelling.

These fissures in the incisivi I also found in the premolars. They also have a vascularized enamel-organ.

Fig. 5 represents a premolar from the upper-jaw. Here again we observe retraction through shrivelling. The odontoblasts have scarcely started the formation of the dentine. Only an extremely thin layer lines these cells. The vessels penetrate from the enclosing connective tissue into the enamel-organ, but they never reach the stratum intermedium, as was the case with the incisivi. There is always a layer of stellate reticulum between the

capillaries and the stratum intermedium. But here also the vessels cross and recross the pulpa, being most numerous between the folds of the molars.



Fig. 4.



Fig. 5.

Another peculiarity is shown by this picture. There is namely an enamel-niche in the premolars of the upperjaw and in all the molars. In the false molars they have already developed into tunnels.

Whereas in the incisivi and the premolars the enamel-organ was completely developed, this appears not to be the case with the molars. The odontoblasts just begin to arrange themselves and as yet there is no question about dentinification. Neither has the enamel-pulp been differentiated, while the external epithelium is cubic. We do not meet with vascularization here either. I was going to say *as yet* for I am convinced that the bloodvessels enter into the enamel-organ only during the development. The incisivi that began to develop first, display a vascularization that is furthest advanced. The vessels close up to the cells of the stratum intermedium. In the premolars this process has not yet advanced so far. Here the vascularization is of a more recent date, whereas in the molars no bloodvessels have at all entered into the enamel-organ.

Amelogenesis begins only after the bloodvessels have penetrated far into the pulpa. Dentinification, however, begins about simultaneously with the entrance of the vessels, and has proceeded far when the vessels have reached the stratum intermedium.

But why do the bloodvessels enter into the enamel-organ at so early a stage? If what Bolk says, is true, viz. that in non-vascularized enamel-organs the enamel-pulp is intended for the nourishment of the ameloblasts, of which I am convinced, I am bound to conclude that in the case of vascularization of the enamel-organ the pulpa can no longer meet the demands of this nutrition, and that this higher function is assumed by the

bloodvessels themselves. If the enamel-organ is not vascularized, the bloodvessels come into contact with the ameloblasts only after reduction of the pulpa, but according as the demands of this nutrition increase, the bloodvessels are sure to assume sooner the task of the pulpa, i.e. they enter into the enamel-organ.

In *Dactilomys* we saw that amelogenesis occurs only after the bloodvessels have reached the stratum intermedium. So the bloodvessels grow into the enamel-organ at an early stage, and take over the nutrition of the ameloblasts. But I insist on it that here also the ameloblasts are fed first by the enamel-pulp and only later on by the bloodvessels.

#### LITERATURE.

- W. H. F. ADDISON and J. L. APPLETON Jr. The vascularity of the Enamel-organ in the Developing Molar of the Albino-Rat. *The Am. Journ. of Anat.* Vol. 31, Nov. 1922.
- L. BOLK. Ueber ein Gebiss mit vaskularisierten Schmelz-organen. *Anat. Anz.* Bd. 48. 1915.
- J. N. BROOMELL and PH. FISCHER. *Anatomy and histology of the mouth and teeth.* 5th edition 1917.
- J. TH. CARTER. The Cytomorphosis of the Marsupial Enamel-organ and its Significance in Relation to the structure of the completed Enamel. *Phil. Trans. Roy. Soc. Ser. B.* 208.
- A. HOPEWELL—SMITH and H. W. MARETT TIMS. Tooth-germs in the Wallaby. *Proc. Zool. Soc. London* 1911.
- A. E. JORDAN. Further evidence concerning the function of osteoclasts. Febr. 1921. *Anat. Rec.* Vol. 20.
- A. E. JORDAN. The comparative histology of the enamel organ of the mammalian tooth, with special reference to its blood-supply. *Am. Journ. of Anat.* Vol. 29. 1921.
- E. KLEIN and E. NOBLE SMITH. *Atlas of Histology* 1880.
- CH. LEGROS et E. MAGITOT. Morphologie du follicule dentaire chez les Mammifères. *Journal de l'anat. et de la Phys.* 1879.
- J. HOWARD MUMMERY. The microscopical and general anatomy of the teeth. Second edition 1924.
- ED. PASCALIS. La vascularisation des Follicules dentaires des carnassiers et de l'homme. Thèse 1918.
- F. T. PAUL. The enamel organ. *Dental Record* Vol. 16. 1896.
- EDW. P. POULTON. The true teeth and the horny plates of *Ornithorhynchus*. *Quart. Journ. Micr. sci.* Vol. 29. 1889.
- A. PRENANT. Contribution à l'histogénèse de l'email dentaire. *Arch. de Morph. Gén. et Exp.* 1924.
- WM. G. SKILLEN. A report on the formation of dentin and cementum relative to the structure of the root end. *Journ. Nat. Dent. Assoc.* Vol. 8. 1921.
- C. WEDL. *Pathologie der Zähne* 1870.
- J. T. WILSON and J. P. HILL. Observations on tooth development in *Ornithorhynchus*. *Quart. Journ. Micr. sci.* Vol. 51. Febr. 1907.
-

**Zoology.** — *On the structure, life-history and development of Hedriocystis pellucida* HERTW. & LESS. By H. R. HOOGENRAAD. (Communicated by Prof. J. F. VAN BEMMELEN.)

(Communicated at the meeting of February 26, 1927).

1. *Collection and conservation of Material.*

Those Heliozoa, whose protoplasmic body is enclosed in a homogeneous, spherical skeleton, generally provided with a stalk, and perforated by outlets for the pseudopodia, were in 1874 by R. HERTWIG and E. LESSER united in the group of the *Desmothoraca*. This group contains a very small number of genera, only one of which, the genus *Clathrulina*, has become more accurately known, and of this moreover but one single species *C. elegans* CIENK. It is true that SCHAUDINN in 1896 mentioned a second genus *Orbulinella* ENTZ Sr., as being rescribed with sufficient precision, but the author himself of this genus argues in favour of the view, that it occupies a separate position in the systematic arrangement, in the neighbourhood of the Foraminifera. In the same way PENARD, who however did not know the animal from personal observation, presumes that *Orbulinella* stands nearer to the Thecamoebic Rhizopods than to the Heliozoa.

The species belonging to the genera *Elastera* and *Choanocystis* until now have only been observed by their discoverers, and moreover in a very restricted number of widely spread specimens, so they must be considered as quite insufficiently known and therefore of very doubtful value for systematic arrangement.

Somewhat better stands the case for the genus *Hedriocystis*, which HERTWIG and LESSER proposed in 1874 for a species *H. pellucida*, discovered by them in the neighbourhood of Bonn. Later on PENARD described a second species *H. reticulata*, and afterwards BROWN distinguished a third one: *H. spinifera*.

Yet just like those named above the representatives of this genus are so rare and consequently so incompletely studied, that older investigators (ARCHER, SCHAUDINN, BÜTSCHLI), who were only acquainted with the species *H. pellucida*, considered even this to be a doubtful form, which possibly should be united with *Clathrulina*. As the more recent observations have not added much of importance to these older statements, the only form of the whole group *Desmothoraca*, that might be considered as characterized with sufficient accuracy, remained the species *Clathrulina elegans*.

In September 1926 I was able to collect material from the so called Wisselsche Veen near Epe o/d Veluwe, a locality, which for many years I have



highly valued as a rich emporium for Rhizopods and Heliozoa, amongst which were several of the rarest and most remarkable forms ever known. Of those that I have already mentioned on former occasions, I wish to refer to *Penardia mutabilis*, *Vampyrella lateritia*, *Hyalodiscus rubicundus*, *Nuclearia caulescens*, *Quadrula symmetrica*, *Heleopera petricola*, *Cryptodiffugia compressa*, *Paulinella chromatophora*, *Raphidiophrys elegans* and *viridis*, *Acanthocystis aculeata* and *turfacea*, *Clathrulina elegans*, *Clathrella Foreli*.

In the same way the spoils of this years collecting proved again rich in interesting forms, among which *Biomyxa vagans*, *Raphidiophrys pallida*, *Plagiophrys scutiformis*, *Gromia nigricans*, *Diffugia oviformis*, *Lieberkühnia Wageneri*, were new for the locality, the two latter moreover for the dutch fauna in general.

But what chiefly attracted my attention was the frequent occurrence of a great many specimens of one of the above-mentioned species of Heliozoa, namely *Hedriocystis pellucida* HERTW. & LESS. Since the discoverers, who observed it about 1870 in the neighbourhood of Bonn, this animal had been found by LEVANDER in Finland (1892), and by PENARD in a few localities near Geneva, about 1900, but nearly always in scanty numbers. PENARD is the only one, who gives a somewhat detailed description of its structure and life-habits, but its development has hitherto remained unknown.

The sample of water, in which the animals occurred, was taken from a shallow marshy pool, recently called into existence by the removal of the superficial layer of peat. I studied the animals in the usual coverglass-preparations, but always added a few algae and a little detritus to the liquor, with the view of keeping them as much as possible in natural conditions.

As experience proved, when the evaporated liquor was regularly replaced by distilled water, the animals kept alive for days and even for weeks in the same preparation, and went on propagating. In this way they could be studied at a magnification of 400—800 diameters.

## 2. Structure of the protoplasmic body and the shell.

The protoplasmic body of *Hedriocystis pellucida* (Fig. 1) is, in the same way as that of *Clathrulina elegans*, enveloped by a nearly spherical shell, which is fixed to the fragments of detritus lying in the water, by means of a long and thin stalk. The animals born in my preparations, often fixed this stalk on the coverglass or the slide.

On the surface of the shell conical projections are to be seen, either blunt or pointed, provided at the tip with exceedingly minute openings by which the pseudopodia emerge. I have never succeeded in observing these pores directly, but owing to a circumstance to be described later, I have been able to calculate their width at a diameter of  $\pm 2 \mu$ . By the presence of these protuberances the shell in optic sections has the appearance of a more or less regular polygon with 4, 5, 6 or 7 sides and with prominent angles ;

by elevating the object-lens to a higher level, the eminences on the surface of the shell assume the aspect of sharply defined, dark crescenticform lines, which give a very characteristic appearance to the animal.

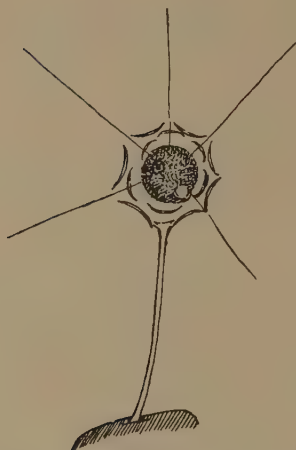


Fig. 1. *Hedriocystis pellucida* HERTW. et LESS. A specimen of the type. In the protoplasmic body at the left topside one sees the nucleus; at the right bottom-side a contractile vacuole.  $\times 800$ .

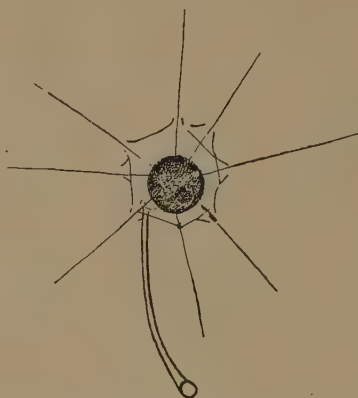


Fig. 2. *Hedriocystis pellucida*. A specimen of the deviating form.  $\times 800$ .

The above-described form represents the type, but besides this my material contains rather a considerable number of specimens of a second form, which though corresponding with the typical one in all other respects, is markedly distinguished from it by the structure of the shell. In this deviating form the thickness of the shell is much less, which makes the contourline of the optic section decidedly slimmer, to such an extent as to reduce it to a hardly visible line. In the second place the protuberances are less pronounced, so much so, that in high-level observation they never produce the above-described effect. The specimens showing this deviation were always completely colourless, those of the type being of a yellowish-red shade. PENARD considers this shade as a hue of the substance of the shell proper, but according to my view it seems more probable that it is caused by a phenomenon of interference. From a careful comparison of descriptions and illustrations I come to the conclusion, that the discoverers HERTWIG and LESSER have only seen what I have just now described as the deviating form <sup>1)</sup>, while the animals observed by PENARD

<sup>1)</sup> They compare the external appearance of the animal to a „Morgensternartige Keule“, meaning a mediaeval club, which at its swollen top is provided with spikes. This description corresponds much better indeed with the variety than with the type.

probably represented the type. Transitions between the two forms never came into my observation.

The diameter of my specimens fluctuated between 20 and 25  $\mu$ , their dimensions therefore correspond with that of the animals studied by former investigators.

As already remarked by PENARD the stalk is probably solid, not containing a central canal like that of *Clathrulina*. Its length was not in all cases easy to be measured, especially so, when it stood in a more or less oblique position. In the cases, where I was able to take its measure, its length varied between 50 and 90  $\mu$ , its thickness is  $\pm 1 \mu$ . The distal extremity of the stalk is either simply broadened, or split up into a small number of tiny fibres, giving it a more or less rooted appearance, as also sometimes occurs in *Clathrulina*.

As to the material of which the shell and its stalk are composed, nothing can be asserted with certainty; it seems to me to be rather more chitinous than that of *Clathrulina*, which is generally assumed to contain a certain amount of silicon. The protoplasmic body contained in this envelope has an average diameter of no more than 10—12  $\mu$ , and is freely suspended, probably by means of the pseudopodia, in the centre of the shell, or a little excentrically. In outward appearance it looks most like that of *Actinophrys sol*. Generally it shows no very distinct shade, but is tinted somewhat blueish-green. Its form is nearly spherical or somewhat flattened in a spheroidal way.

At its circumference it is sometimes lobed or denticulated in an irregular manner, but habitually it is rather purely rounded. The line of its contour is sharp and dark, which points in the direction of great optic density. Consequently the animals, even when observed with low powers, show a sharply defined appearance, making them immediately conspicuous, when once we have got acquainted with them.

Big food-morsels are generally absent, but small dark granules are always present, though in varying number.

Probably on account of the compact structure of the protoplasm, the nucleus is difficult to be distinguished. In cases where it is visible (Fig. 1, 6), it presents itself as a big, round spot, lying somewhat excentrically and provided with a large light-grey endosome.

Vacuoles seldom occur. Only when an uncommonly big prey has been swallowed (Fig. 6), a vacuole surrounding it can be clearly distinguished. Contractile vacuoles are always present, to the number of two or three. (Fig. 1, 6). They never form prominences on the surface, as in *Actinophrys*; their bulk size is variable and may attain  $\frac{1}{3}$  of the diameter of the protoplasmic body. Their contractions are regular, with usually short periods; in one case where I was able to observe this process for a considerable time, the interval between two successive contractions amounted to  $\pm 30$  seconds (temperature 18°).

Excretion of undigested food-remains was never observed by me.

The pseudopodia are generally present in restricted numbers, in toto probably never more than 10 to 15, but sometimes only 6 to 8. In the normal condition they are thin, straight, unbranched and rather long, up to  $\pm$  four times the diameter of the shell; in most cases they start with a sharply defined, unbroadened base from the body of the protoplast. They are therefore different in several respects from the pseudopodia of *Clathrulina elegans*, which broaden considerably towards their base, and moreover are often forked, and sometimes form anastomoses. Eventually however, especially when food is taken in, the pseudopodia of *Hedriocystis* assume various irregular shapes by curving, broadening and by the appearance of vacuoles (Fig. 6). I never was able to observe a real current of granules; what might easily be considered to be such, in reality turned out to be nothing but the sliding movements of a microbe, absorbed as a food-object, and transported in the direction of the protoplasmic body.

### 3. Life-history.

In my specimens the food, conformally to what was observed by PENARD, chiefly consisted of bacteria, only in a few cases a small Flagellate or the swarm spore of some alga was captured. The absorption occurs in the usual way. The pseudopod, with which the prey comes in contact, generally forms a very spacious food-vacuole around it. Inside this the food-morsel becomes slowly moved towards the shell (Fig. 6). During the ingestion the pseudopod remains stretched: the prey, enclosed in its vacuole, glides along it with tolerable speed. It is clear that this movement points to the existence of a protoplasmic current in a centripetal direction.

Small objects speedily disappear into the interior of the protoplasmic body through the pores of the shell, bigger ones on the contrary remain for a long time fixed on its outside, surrounded by a protoplasmic mass, which itself is in communication with the rest of the body. After a certain lapse of time the prey is seen to become smaller and less sharply defined and at last it disappears completely in the protoplasm. In some cases I could observe that the last remnant slipped into the shell and sank into the protoplasmic body. Just as in *Clathrulina* part of the food at least is digested outside the shell.

Under normal circumstances the animals swallow rather a large quantity of food. It is not rare that several pseudopodia at a time are seen to carry bacteria towards the protoplasmic body. Digestion and absorption of the food seem to take place rapidly; occasionally I succeeded in following rather big food-morsels from the moment of their entrance into the protoplast up to that of their disappearance or becoming unrecognizable; the whole of this process lasted a couple of hours. By this rich nutrition the young animals grow rather rapidly: in two or three days they attained the normal size, and shortly afterwards the reproduction sets in.

As far as is known up till now, this latter always occurs by binary fission of the protoplasmic body; one of the daughter-individuals then escaping from the shell, to form a new specimen, which provides itself with a stalk, while the other remains behind, occupying the old shell. The processes of division and formation of the shell together take less than twelve hours; after this both animals enter into the period of food-ingestion and growth, then a new division sets in and so on. So the whole of the simple life-history of *Hedriocystis* takes place within three days, of which the greater part is devoted to feeding and growing, a much smaller period to propagation. The latter is — as far as known — purely vegetative, processes of impregnation or sexual differentiation have hitherto remained quite unknown for this animal, as for most Heliozoa in general.

#### 4. *Reproduction.*

The older investigators do not seem to have observed the division. It is true that HERTWIG and LESSER, as well as PENARD, make mention of specimens, in which two protoplasmic bodies were seen inside one shell, and which therefore undoubtedly had just before passed through the act of division, but about this process itself they tell us nothing. I have been able to follow it several times from beginning to end; as far as I could see, it always takes the same course.

The division (Fig. 3) takes place during activity inside the shell, which



Fig. 3. *Hedriocystis pellucida*. Divided specimen of the deviating form.  $\times 800$ .

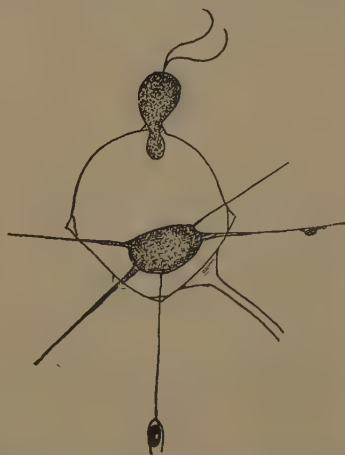


Fig. 4. *Hedriocystis pellucida*. Divided specimen of the deviating form, one of the daughter-individuals leaves the shell as a swarm spore with two flagella. The other animal is taking in food with two of its pseudopodia'  $\times 1200$ .



means, that the pseudopodia remain protruded. Nor is there the slightest indication of encystment or of a resting stage. I never saw cysts, as described by HERTWIG and LESSER and figured by PENARD; and I am inclined to explain their observations as simply caused by temporary retraction of the pseudopodia, a condition which I have observed more than once. PENARD himself states that the wall of the cyst was "lisse et peu apparente"; HERTWIG and LESSER express themselves in the same sense.

The divisional process proper shows nothing particular; it is completed in a few minutes. The behaviour of the nucleus could not be observed. During the division the capture and absorption of food goes on as usual.

The plane of division generally stands perpendicularly to the stalk, or at least to that part of it, which is in immediate contact with the shell.

After division is complete, the two daughter-individuals generally change their position, and lose their definite orientation inside the shell-cavity. They then assume a broad-elliptic shape, their long axes standing parallel, while they are a little flattened on the side where they are in contact with each other.

For how long the animals remain in this condition, I cannot tell, but I presume it is only for a short time, and certainly no more than 12 hours. At the end of this period one of the two individuals loses its rounded shape and begins to make amoeboid movements, at first at intervals, afterwards more continually. Shortly afterwards two flagella become visible, arising side by side on the surface of the body and soon they begin to show slow movements. They are exceedingly thin and therefore difficult to be seen, probably also because their optic density is but slightly different from that of water. Shortly after this, the pseudopodia are retracted, and the animal begins to force itself through one of the shell-pores (Fig. 4). At first a small drop of hyaline plasma appears on the external surface of the shell, in the course of time this slowly enlarges by continuous outpouring of plasma through the shell-opening; the part of the body still remaining inside the shell proportionately diminishing in bulk. During this act the animal is continually constricted by the extremely narrow shell-pore, and so has the form of a dumb-bell; the diameter of the constriction amounts to  $2\ \mu$ ; therefore this will probably be the width of the shell-pore. From this extreme constriction, and the fact that its diameter remains unchanged during the whole act of escaping from the shell, we may conclude that the animal does not succeed in enlarging the pore through which it forces itself out; the shell therefore, however thin, must be of a rather rigid texture.

When the last part of the protoplast is still inside the shell, the two flagella have already passed out and their movements, at first slow, grow faster, and under their influence the free part of the animal, still fixed by the shell-pore, is brought into a rapid vibration. Soon afterwards the animal begins to rotate along its longitudinal axis, and thereby the last contact with the shell is broken. The animal then stretches and shortens itself and swims away as a zoospore with peculiar undulating and rotatory

movements. The other half, remaining in the shell, rounds itself and takes up a central position, so assuming the normal state for this animal.

The zoöspore (Fig. 5) has a cylindrical shape with rounded ends, sometimes the posterior pole is a little pointed. Its length is  $15\ \mu$ , its breadth  $3\ \mu$ . It is transparent, uncoloured and rather regularly granulated. In its anterior part is a nucleus, somewhat difficult to discern and at the posterior end a small but distinct contractile vacuole. The length of the flagella is probably somewhat greater than that of the spore. The rhythm of the swimming movements is so characteristic, that even with a low power it is easy to distinguish the zoöspore from similar organisms.

After a short time, usually only two or three minutes, the movements become slower, and soon the zoöspore fixes itself, and rounds itself off as a protoplasmic body of  $\pm 10\ \mu$  in diameter.



Fig. 5. *Hedriocystis pellucida*. Swarm spore. Before the nucleus, behind the contractile vacuole.  $\times 1200$ .

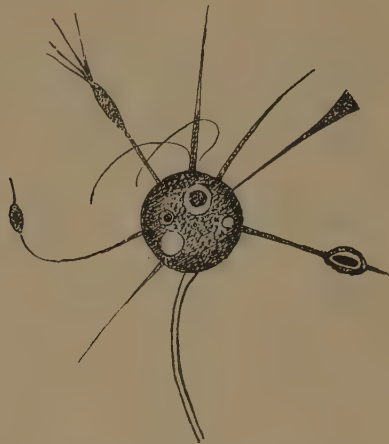


Fig. 6. *Hedriocystis pellucida*. Young specimen before the formation of the shell. At the top two flagella are still visible, at the bottom the outgrowing stalk. Abnormal pseudopodia forms two of the pseudopodia with a captured microbe. In the plasmic body at the top the nucleus, at the bottom, right and left side a contractile vacuole, at the left of the centre a digestion vacuole with a microbe.  $\times 1500$ .

The flagella, which, during the free-swimming period, remained invisible, on account of the rapidity of their movements, now become visible again. During a considerable lapse of time, at least a couple of hours, they execute peculiar, pendulating movements, but at last they disappear. What may be their final fate, to become dejected, or retracted, I am not sure; their

extraordinary thinness already necessitates to the greatest exertion to get a look at them, and even then one only succeeds from time to time just for a moment. Almost immediately afterwards the pseudopodia make their appearance, within a minute or so they have attained their normal length. In its general aspect the animal now rather resembles a miniature specimen of *Actinophrys sol* (Fig. 6). During the first period of their existence the pseudopodia appear to be in a very unstable condition; they often show deviations from the normal, straight and thin form which afterwards is typical for them, these consist in enlargements, sinuosities, ramifications and the formation of vacuoles. While the flagella are still present, the taking in of food already begins. Likewise the development of the stalk is a question of a few minutes only. In the same way as described by HERTWIG and LESSER for *Clathrulina* the stalk is formed by a protoplasmic protrusion of the body. The supposition of PENARD, that originally the stalk is nothing but a pseudopodium, as far as I can see may very well be correct. Be this as it may, soon afterwards the stalk is distinguished from a common pseudopodium by its superior thickness and stabile diameter. Originally it stands rather strictly perpendicular to the surface on which the stalk fixed itself, probably in consequence of a thigmotactic stimulus. This may be stated with special ease when the support on which the organism fixes itself is the coverglass or the slide. The position of the stalk then being vertical, it becomes visible in projection as a sharply drawn ring of very small diameter, either above or under the protoplasmic body. By means of the scale on the micrometric screw the length of the stalk could be very accurately measured. Still easier could this be done by direct measuring, when the stalk was fixed to some object floating in the water, and thereby was occupying a horizontal position. After fixation the stalk elongates considerably just as in *Clathrulina*; after half an hour it attains its definite length. But long afterwards, for at least two hours, the upper part of the stalk, adjoining the protoplasmic body, shows amoeboid movements in the shape of blunt protuberances, which soon flatten out again. In a few instances I also was able to distinguish a current of granules in this part of the stalk, of the same character as always occur in the body. In the young stalk of *Clathrulina* HERTWIG and LESSER mention also such a granular current. All these facts plead distinctly for a genetic connection between stalk and pseudopodia, though perhaps the assertion that the stalk originally becomes formed as a true pseudopodium may not be quite accurate. In a phylogenetic sense however the view that the stalk of *Clathrulina* and *Hedriocystis* must be derived from a pseudopodium, seems to find support in the above-mentioned facts. A few hours after its formation the stalk begins to change its aspect; it becomes thinner and more transparent, the amoeboid motility disappears, probably on account of a change of its purely protoplasmic nature to the harder, more chitinous material, of which the definitive stalk consists.

An insoluble problem in this respect is the question, how the connection

of stalk and shell is brought about. For though the stalk undoubtedly is formed as a product of the protoplast, it afterwards has lost all ties with it, and passes exclusively into the shell. As far as I can judge, this question is not even completely solved for *Clathrulina*, though here it applies somewhat differently, because in *Clathrulina* the stalk contains a canal. It is true that also this stalk is originally formed as a purely protoplasmic stem, but then its outer layer hardens to a tube corresponding with the skeleton of the protoplasmic body, and, as HERTWIG and LESSER take it, the protoplasmic core then withdraws from the lumen.

In this connection we also may call to memory PENARD's communications, that in *Nuclearia caulescens* PEN., an organism of doubtful affinities but most probably connected with the Heliozoa<sup>1)</sup>, both types of stalk seem to occur. One of them corresponds according to PENARD's description with the formation of the stalk in *Hedriocystis*.

About the formation of the shell I can make only a few remarks; nor is there much known about it in *Clathrulina*. As in this latter form the shell is always formed later than the stalk; HERTWIG and LESSER even saw a specimen dividing before the shell was formed, but I never met with this phenomenon.

At the moment the outline of the shell first becomes visible as an extraordinary thin stria, we should expect it to show itself immediately on the surface of the protoplast; this however is not the case, but it is seen from the beginning at the normal distance from this surface. This means that it is separated from it by a space amounting in diameter to that of the protoplast itself. The shell therefore seems to be formed independently of the body, so to say as a coagulum in the empty space. Up till now I cannot give an explanation of this curious appearance.

### 5. The function of the zoöspores in Heliozoa.

As we saw in the foregoing, *Hedriocystis pellucida* exclusively multiplies by fission — at least as far as we know up till now — and this division is followed by the formation of a zoöspore by one of the daughter-individuals. This mode of development likewise occurs in *Mikrogromia socialis* R. HERTW. This animal also splits up into two halves, of which one leaves the shell of the mother-individual, but in this case it may either assume the form of a zoöspore provided with two flagella, or that of an *Actinophrys*-like organism. The remaining history of these stages however is totally unknown.

In *Hedriocystis* the same process repeats itself periodically with one and the same animal, which thereby assumes the character of a mother-organism, producing a whole generation of daughter-individuals by serial division.

---

<sup>1)</sup> This organism was found by me on former occasions already, in a few specimens in the „Wisselsche Veen“.

The division causes the multiplication of individuals, the formation of the zoöspore favours the distribution of these individuals in space.

Both processes: binary division and formation of zoöspores, also occur in *Clathrulina*, though as far as we know the connection between them is less regular.

The alliance between this and *Hedriocystis*, which originally was only inferred from the structure of the shell and the protoplast, therefore finds a new support in development.

So we may state that the group of *Heliozoa desmothoraca* contains at least two forms, *Clathrulina elegans* and *Hedriocystis pellucida*, which in their structure and development are fairly well-known. The rest of the species brought under that heading, remain rather uncertain.

In the group of *Heliozoa*, apart from *Clathrulina elegans*, zoöspores had already in former days been observed in the case of *Acanthocystis*-species. This mode of reproduction likewise frequently occurs in other groups of Rhizopods, especially with Radiolarians. PASCHER, in his well-known treatise on the connections between Flagellates and Rhizopods, combates, especially on botanical grounds, the opinion of DOFLEIN, that in all departments of Rhizopods the forms with flagellate phases are the most primitive.

Now the systematic place of the *Heliozoa* as a whole is still rather uncertain, and the same is the case with the different smaller units which compose it, with regard to their mutual relationships; yet we may assume with a fair amount of probability that the *Desmothoraca*, to which *Clathrulina* and *Hedriocystis* belong, with their peculiar perforated and pedunculated shells, represent a less original condition than the naked *Aphrothoraca*, such as *Actinosphaerium*, and *Actinophrys*. The occurrence of zoöspores in the more highly differentiated types and their absence in the lower constitutes, as regards *Heliozoa*, an argument in favour of PASCHER's views.

PASCHER, who, in the above-cited study, proposes the thesis, that Flagellates are more primitive organisms, Rhizopods on the contrary more secondarily modified ones, affords a great phyletic value to zoöspores, in the sense of the theory of recapitulation.

Zoöspores should be temporary recollections of more primitive stages of development, through which the organism passed in former periods. Even in relatively high-differentiated forms they always still occur, be it as carriers of the sexual function, or as emigrants for the distribution of the species.

Even if this supposition should prove right, it affords no explanation of the fact, that the zoöspores, as reminiscences of more primitive flagellate organisms, may occur in higher specialised forms, while they have disappeared in less specialised, more original types. Their occurrence with sedentary organisms, as *Clathrulina* and *Hedriocystis*, where nothing is known about a possible sexual function for them, might however, as



PASCHER points out, be brought into connection with a chance of distribution in space, beyond the immediate neighbourhood of the mother-organism.

#### LITERATURE.

1. L. CIENKOWSKI, Ueber die *Clathrulina*, eine neue Actinophryen-Gattung (Arch. f. mikr. Anat. **3**, 1867).
2. R. HERTWIG und E. LESSER, Ueber Rhizopoden und denselben nahestehenden Organismen (Arch. f. mikr. Anat. **10**, 1874 Supplementheft).
3. R. HERTWIG, Ueber *Mikrogromia socialis* etc. (Arch. f. mikr. Anat. **10**, 1874, Supplementheft).
4. G. ENTZ Sr., Ueber die Rhizopoden des Salzteiches von Szamosfalva (Termes. Füzetek **1**, 1877).
5. J. LEIDY, Freshwater Rhizopoda of North America (U. S. Geol. Survey 1879).
6. S. M. FOULKE, Some Phenomena in the Life-history of *Clathrulina elegans* (Proc. Acad. Nat. Sc. Philadelphia, 1884).
7. F. SCHAUDINN, Heliozoa (in: Das Tierreich) 1896.
8. F. SCHAUDINN, Ueber das Centralkorn der Heliozoen etc. (Verh. Deutsch. Zool. Gesellsch. Bonn, 1896).
9. A. LANG, Lehrbuch der vergl. Anat. d. Wirbellosen Tiere. I. Protozoa. 1901.
10. E. PENARD, Les Héliozoaires d'eau douce. 1904.
11. F. DOFLEIN, Lehrbuch der Protozoenkunde. 2te Aufl. 1909.
12. A. PASCHER, Flagellaten und Rhizopoden etc. (Arch. f. Protistenkunde **38**, 1917).

Deventer, December 1926.

---

**Anthropology.** — *On the Origin of human Races.* By Prof. L. BOLK.

(Communicated at the meeting of March 26, 1927).

In the anthropological literature many systematic divisions of mankind into races may be found. Now it strikes us that so little has been done concerning a systematic study of the origin of the human races. Leaving out of consideration the memoirs in which problems of general biology are treated, e.g. whether the origin of the races was a monogenetic or a polygenetic one, or whether they are not rather to be considered as subspecies, and suchlike questions, there is until a few years back scarcely any indication of a trial to explain the origin of races from a fundamental point of view. Of course, there are plenty of vague generalities, which can invariably be reduced to the little satisfactory standpoint occupied by DARWIN, when he summed up his opinion about the origin of species about as follows: "We may conclude that the variations giving origin to the human races, have been brought about either directly by exposure to various conditions of life, or indirectly by some form or other of selection". At the close of the chapter treating on the origin of races, in his "Descent of Man" he also admits the possibility that races may have evolved from spontaneous variations.

It is easy to understand that such generalities cannot furnish a basis for a methodical interpretation of even the most conspicuous somatic properties of the different races. DARWIN himself tries to do so with respect to the black skin of the negroes (to which I shall revert) but finally he declares that not one interpretation satisfies him.

• In recent years, however, the problem of the origin of races has been studied on more sharply defined biological principles than those produced by DARWIN's theory. Starting from the consideration that the morphogenesis of the individual is regulated by the hormones of the endocrine system, now one is trying to retrace the somatic differences between the groups of individuals forming together one race, to a different agency of these substances. This reasoning is quite logical, and KEITH has been the first, who, in 1919, viewed the typical physical characters of the various races from that standpoint. After him several German authors have advocated the same opinion. The different hormones are supposed to be in a certain condition of equilibrium, a change in this equilibrium revealing itself in a more or less strong development of a somatic character. So every race would as PFUHL puts it, have its own type of *hormonic equilibrium*.

This idea, first unfolded by KEITH<sup>1)</sup>, as stated above, has already induced other investigators to study the degree of development of the endocrine glands in different races; and as yet some positive results have been obtained.

In their speculations on the origin of the human races I side with these authors the more readily as they are quite within the sphere of my own conceptions as to the origin of man. But, I'll go a step further, and not confine myself to the pronouncement that the racial qualities are indicative of a difference in, what I would term "the hormonistic index" of the individuals, but endeavour to show in what way certain racial characters result from these differences.

On the face of it this looks like a difficult task, yet it will appear that along this path it will be possible to comprehend the origin of racial qualities and consequently the origin of the races.

In my exposition I start from the hypothesis (in my opinion irrefutable), that the same cause from which arose the specific physical characters of mankind as a whole, also brought about the specific properties of the races, into which the genus differentiated. This, then, points to a common prime cause for the genus and for the races.

Some time ago I published a résumé of the principles of my theory of Anthropogenesis<sup>2)</sup>. The leading thought of it was that man resulted from a retardation in the developmental tempo, and the course of life of the organism. Human life proceeds, to put it figuratively, like a retarded film. Now that retardation was, as regards the somatic development, not a regular one: certain physical characters were restrained more intensely in their evolution than the whole, of some the development was even completely arrested, i.e. suppressed. The consequence of this was that the form became characterized by the possession of properties of a fetal nature. This is what I termed the fetalisation-theory of our morphogenesis.

Now the study of racial features confirms the correctness of this theory, as it appears that just as the specific physical qualities of man are in general persisting fetal conditions, so also the specific racial features bear completely this character. In some sense, therefore, a race of men may be defined as a group of individuals in whom a definite somatic property or condition terminates its evolution at a certain stage, whereas in all other individuals its evolution is still proceeding till the very end. In other words: the specific racial features are fetalisation phenomena. The connection between this conclusion and the conception, just now alluded to, of the racial characteristics as manifestations of a different equilibrium of the hormones, is not far to seek. For, already in my discourse, just now referred to, I pointed out that the fetalisation

<sup>1)</sup> The differentiation of mankind into racial types. *The Lancet* 1919, Vol. 2.

<sup>2)</sup> On the Problem of Anthropogenesis. *Proceedings Kon. Akad. v. Wetensch.*, Vol. 29, 1926.

of the human body, as a whole, results from a checking agency emanating from the hormones. Racial characteristics, then, are merely manifestations of specific checks at work in the evolution of definite groups of men.

It is out of place here to fully discuss this principle, I must confine myself to unfolding the biological principle of the origin of races, and am now going to illustrate this by three of the most striking racial properties, viz. the specific characteristic of the Mongolian race, the colour of the skin, and the shape of the skull.

The so-called mongol-fold or Epicanthus is no doubt the most characterising feature of the Mongolian race: a skin-fold that, starting from the upper eye-lid, runs downward, and bridges over the inner angle of the palpebral fissure. This, however, is but one characteristic of the Mongolian physiognomy. When comparing the structure of the upper half of the face of a Mongol with that of a European, it will appear that the real differences extend farther, and that e.g. the face of a Japanese is typified by what I will designate by the comprehensive name of "Mongol-complex". This complex presents three anatomical peculiarities, viz. the above-named fold, the flattened nose-bridge, and the protrusion of the eye-balls. First let us dwell on the last two.

As ADACHI's investigations have borne out, the topography of the eye-ball of the adult Japanese, relative to the orbit, differs from that of the European: The deep-set eye, often met with among Europeans, does not occur with the Mongolian race. On the contrary, the eye-ball of the Mongol lies protruded to such a degree that nearly the half of it lies before the plane of entrance of the orbit, whereas with the European the eye-ball lies entirely within the orbit or at most for one third out of it. This fact, in conjunction with the flattened nosebridge, enables the Japanese and the Chinese to let a stick rest on both their eye-balls while it passes right across their nosebridge. This anatomical peculiarity is as typical of the Mongol as the occurrence of the epicanthus. Now, if we examine on this head the fetus of the European, it appears that at one stage of his evolution a transitory condition occurs like that which is characteristic of the adult Mongol. In the development of the European we also meet with a stage with flattened nosebridge and prominent eye-balls. But while with the Mongols this stage has become the terminal condition, it is for the European a transitional phase; for during further development the eyeball sinks deeper into the orbit, and the upper part of the nosebridge becomes prominent. Thus we see that with the Mongol these two components of the "mongol-complex" are persisting fetal conditions.

The same holds good for the third component of the complex, the epicanthus, or the mongolian fold. In order to demonstrate this, we have to make a short exposition of the development of the system of folds round the human palpebral fissure.

The different phases of this development after my personal investigations

are represented by figs. 1 to 7. When the eyelids have been formed and cover the considerably prominent eyeball, there exists a marginal groove compassing the eyeball medially and upwards, while the medial angle of the

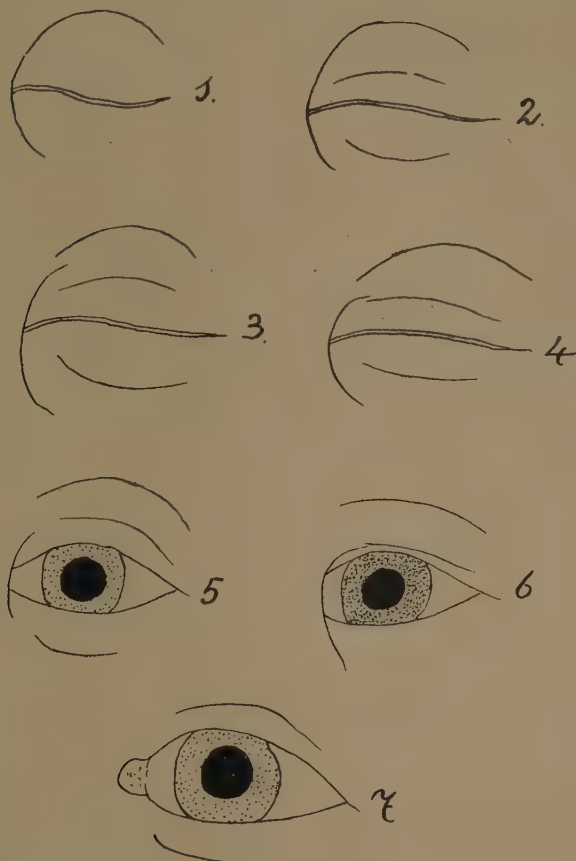


Fig. 1—7.

palpebral fissure extends to this groove (Fig. 1). After this a skinfold is evolved on the upper and the lower palpebra, which, however, does not reach the groove (Fig. 2). Subsequently the marginal fold splits up into two fragments, a superior and a medial one. Henceforth the latter forms a skinfold by itself, in the middle of which the medial end of the palpebral fissure terminates (Fig. 3). This medial fold slightly overlaps the nasal end of this fissure, so that in this phase the fetus has a true epicanthus (Fig. 4). Now, in case the eyes were to open in this stage of development, we should see a condition as illustrated in Fig. 5, in which the



medial canthus is overarched by the medial fold or epicanthus. It is commonly known that in neonati of the European this fold occurs occasionally, when the normal process of development, in which the epicanthus disappears, has been retarded. In normal cases this disappearance takes place in the European fetus during intrauterine life already, and the condition of fig. 7 is brought about. In the Mongolian race, however, the fetal epicanthus persists, and unites with the secondary skinfold on the upper eyelid (fig. 6). It is evident, therefore, that epicanthus and mongolian fold are not completely identical: the mongolian fold has originated from a combination of the fetal epicanthus with the secondary fold of the upper palpebra.

This brief description of the development of the skinfolds round the palpebral fissure goes to show that the epicanthus of the Mongols is a persisting fetal property.

The embryological investigation of the development of the system of folds round the palpebral fissure indicates that an epicanthus normally occurs during intrauterine life of Man. However, while as a rule this stage is transitory and the epicanthus disappears, the fetal condition persists in the Mongolian race. The "mongol-complex", as I termed it, which differentiates the Mongolian race so sharply from the other races, appears, indeed, to be a veritable fetalisation phenomenon. So with respect to this feature an interrelation exists between the races, corresponding to the relation of the general human characteristics to that of the other primates. Conditions, which in Primates are generally transitional, embryological stages, have become terminal stages in Man. It is just the same with the epicanthus: a condition, which in general is a transitional stage in the development of Man, has become a terminal stage with a definite race.

The fact that in every man the system of folds passes through an epicanthus-stage, elucidates two phenomena that I wish to accentuate here. Firstly the occurrence, already mentioned, of an epicanthus in neonati of the white race. In most cases this fold gradually disappears in the first years after birth. It is evident that the occurrence of such a fold is not at all due to some hereditary influence of a hypothetical Mongolian ancestor, as was formerly supposed; it is merely a pathological retardation of a normal evolutionary process. Occasionally this retardation gets so intense, that the fold persists also in Europeans, in this case the retardation has acquired a pathological character. This is one of the instances pointing to the fact that stabilization of a fetal condition being physiological for one race, may be pathological for another.

The second instance I wish to call attention to, and which becomes quite comprehensible to us now, is the frequent occurrence of an epicanthus among tribes of the negro-population of Africa. Until recently this was known only for the Hottentots, about 70 % of them having a more

or less pronounced epicanthus. This led to the supposition of a hereditary relationship between Mongols and Hottentots, to which, however, objections have been raised. Another interpretation was brought forward, viz. that while racial characteristics may originate as spontaneous variations, the occurrence of an epicanthus in Mongols and Hottentots was evidence that such variations may arise independently among various groups of men. Subsequent reports stated that in Africa the epicanthus is not restricted to the Hottentots. It is especially among the negroes of Nigeria that it seems to occur frequently.<sup>1)</sup> And the first reporter, convinced that an epicanthus was typical of the Mongolian race, expressed his surprise at discovering Mongolian influence among the population of Central Africa.

Now the fetalisation-theory throws quite a different light upon the occurrence of the epicanthus among the inhabitants of Africa. The persistence of a fetal condition in human ontogenesis, characteristic of the population of Eastern Asia, also takes place frequently among certain tribes of the negro-group, as it does in individuals of the white races.

This can the more easily be understood, as in this case, just as with any other somatic condition, we have not to do with an acquired property. All human individuals have had an epicanthus; with most of them it disappears, with a minority it persists and now becomes a distinctive property of races. The problem why this stabilization occurs in definite groups, we are unable to solve. The retardation, and the fetalisation as phenomena and as evolutionary factors, are easy to demonstrate, but the selection exercised by the retardation we do not understand: it remains a puzzle.

For (and now I am going to discuss the second racial character) why has the pigmentation of the skin, the eyes, and the hair been retarded, even suppressed in a group of men? Am I justified in asking this question? Yes, for the ancestors of Man were black-haired, and black-skinned, the fair type issued from the dark. I must adduce my argument for this assertion.

With regard to the colour of the skin DARWIN takes a somewhat peculiar standpoint. He does not ask: what caused the difference in the colour of the skin, the eye and the hair, but sets himself the question: how is it that negroes have become black? and considers several possibilities, none of which, however, satisfied him.

I am of a different opinion; our ancestor was black-haired and black-skinned, just as the Gorilla and the Chimpanzee. My reason for this assertion is obvious. The genus *Homo*, as a whole, is characterized by black hair and brown to black eyes. Only the *Homo nordicus* possesses fair hair and blue eyes, a property acquired only in a later phase of Anthropogenesis. If, then, black hair and black eyes are characteristic of

---

<sup>1)</sup> SELIGMAN, A Pseudo-Mongolian Type in Negro-Africa. *Man*. Dec. 1924.

Man, we may conclude that also the skin was black, before it lost its hair. With some races this colour has persisted: Negroes, Papuans, Australians. The process of depigmentation was as follows: first the pigment in the skin was lessened: Mongolians, Malays; and afterwards the development was suppressed to such a degree that races with a fair skin were generated, but still in possession of black hair and dark eyes. Their representatives are the mediterranean, and the alpine races. In one race the depigmentation became complete, viz. the nordic race. In this race also the hair and the eyes lost their pigment, the eye became blue, the hair fair.

Now, if we consider the question: What is the nature of this process of depigmentation, causing a racial differentiation of Mankind to such a degree that it became the basis of the first classification, that of BLUMENTHAL? Evidently here also we have to do with different stages of a fetalisation-phenomenon. The faculty of producing pigment, so strong at the beginning that the neonatus was born with the intensely black colour of skin, hair, and eyes of his parents, was trigged in its development, and became less intense. This signified already an approach to the fetal condition, in which the skin is fair, the eyes blue, and the hair light. At first this retardation was limited to the pigmentation of the skin. The negroes show the first indication of it, the neonatus of this race being often of a considerably lighter colour than its parents, and acquiring its definitive colour some time after birth, at most after 6 months, according to records in the literature.

Now, the more the development of pigment in the skin is suppressed, the more the individual will approach the fetal condition: the colourless skin. In the Nordic race this condition persists most completely. In this race, furthermore, the depigmentation is progressive, iris and hair lose their dark colour as a manifestation of a fetalisation-phenomenon. This process of depigmentation is of just the same nature as that in the skin: the production of pigment is suppressed and retarded. Owing to the first circumstance the eyes and the hair become lighter, and as to the latter, we still observe its action in the well-known fact that with fair-haired children as a rule the hair gets darker after birth, a process varying individually considerably. This darkening of the hair and the eyes after birth seldom stays away in children of the fair race, which phenomenon is identical with the postnatal darkening of the skin in negro-children.

So, as to the colour of the skin, the eyes and the hair, the *Homo nordicus* appears to be more fetalized than any other race.

As the third racial character presenting itself as fetalisation-phenomenon, the form of the skull may be discussed. In this paper, in which only principal points can be discussed I shall confine myself for the sake of brevity to the two principal shapes of the skull; the brachycephalic and the dolichocephalic. There are brachycephalic and dolichocephalic races.

By some authors the cause of this difference is considered to be a telluric one, in such a sense namely that mountainous regions favour the evolution of a brachycephalic skull. However, the instances, at variance with this presumed correlation between environment and shape of the skull, are so many that this hypothesis may be repudiated as being faulty.

It was supposed by DARWIN that the increase of stature without a corresponding augmentation of the size of the brain, has provided some races with a dolichocephalic skull. I quite agree with DARWIN in supposing a correlation between increase of stature and dolichocephaly, but to my mind the additional factor, viz. the absence of a coinciding increase of the cerebrum, is superfluous.

However, the preliminary point we have to consider, is again: the shape of the skull of primitive man. This problem is rather difficult, because morphologically the term "primitive man" does not convey a distinct conception. There are no morphological criteria to refer a definite form still to the anthropoids, and to call another form Homo. This may be illustrated by the Pithecanthropus, which nobody will class as a primitive man. Yet, if of this form the calvarium were not known, and only the teeth had come into our possession, we should not hesitate in diagnosing them as the teeth of a primitive man.

When considering, however, that the skull of the Pithecanthropus, and likewise all skulls of the Homo neanderthalensis, known to us, and also the Talgay-skull, are dolichocephalic, while furthermore a brachycephalic skull appears only in a later period, it is within the bounds of probability that primitive man was dolichocephalic. Until recently one might have argued against this supposition that anthropoids are brachycephalic. This objection, however, has become invalid, since the existence of a dolichocephalic race of Gorilla was demonstrated<sup>1)</sup>. The rule, however, that Anthropoids are brachycephalic, suggests the supposition, that the ancestor of Man also possessed a round skull, which during anthropogenesis was metamorphosed into a dolichocephalic one. When tracing the cause of this morphological change, it seems to me that a fair point of issue may be found in DARWIN's idea (just alluded to) that augmentation of body-length may be correlated with the development of dolichocephaly. Although no evidence can be adduced, it may still be assumed on the basis of several considerations, that anthropogenesis was attended with an increase of stature. This increase concerned particularly the trunk and the legs. The connection of this phenomenon with the development of dolichocephaly I imagine to be the following: The increase of the length of the trunk results from a prolonged persistence of the epiphyseal disks of the vertebrae, which is to be considered as a symptom of the

<sup>1)</sup> L. BOLK: "On the Existence of a Dolichocephalic Race of Gorilla. Proc. Kon. Akad. v. Wet., Amsterdam, Vol. 28, 1925.

H. A. HARRIS: Endocranial Forms of Gorilla skulls. American Journal of Physical Anthropology, Vol. 9, 1926.

retardation of the development in general. This prolonged function of the epiphyseal disks caused a vertical enlarging of the vertebrae, and brought about an increase in length of the trunk. But also the epiphyseal disk in the base of the skull persisted and functionated longer, so that the base of the skull was elongated. Add to this the rather considerable enlargement of the cerebellum by which the Os occipitale got protracted in sagittal direction, then we have two possible factors that render the transformation of brachycephaly into dolichocephaly, and the dolichocephalic skull of primitive man, comprehensible.

Now, how did in a later period the brachycephalic skull issue from this form? It would seem to me, that here we have to do again with a fetalisation phenomenon. It should be remembered that every man is brachycephalic during a rather long phase of his fetal life. According to the investigations by SCHULTZ (*Fetal Growth of Man and other Primates. Quaternal Review of Biology, Vol. I*) no dolichocephalic skulls are met with until the 6<sup>th</sup> month of the fetal growth, and only during the sixth or the seventh month the index cephalicus may be lowered so much that the skull has become dolichocephalic. Dolichocephaly, therefore, is a secondary form taking its origin during fetal growth. In brachycephalic individuals this transformation of the skull is suppressed, and the primary fetal form is retained. If we correlate this fact with brachycephaly as racial feature, we come to the conclusion that this characteristic is also due to the persistence of the cranium (now as a hereditary phenomenon) in the form that is typical of the first period of human fetal life. It follows, then, that brachycephaly is also a phenomenon of fetalisation.

It has thus been demonstrated, that there is one and the same cause for the origin of racial features and the genesis of the actual form of Man, as a whole.

---



**Anatomy.** — *A comparison of the form of the callosum and septum in Chinese, Philippino- and Dutch brains.* By Dr. MA (WEN CHAO), Peking. (Communicated by C. U. ARIËNS KAPPERS).

(Communicated at the meeting of February 26, 1927).

The observations in this paper are limited to certain features presented by the corpus callosum and the septum pellucidum in 65 human brains. Of these, 30 were Dutch<sup>1)</sup>, 20 Chinese (Peking) and 15 Philippino (Manilla). All the brains were in good condition. They were cut in the median sagittal plane and tracings of the callosa and fornices were made on thin paper put on the callosum with a sheath of glas between.

The shape of the median sagittal sections of the corpora callosa and septa is seen in Fig. 1, 2 and 3.

I shall first deal with the form of the corpus callosum, discussing subsequently the three parts, in which it is generally divided: the knee (genu), the body (truncus) and the tail (splenium).

The *knee* comprises approximately the anterior fifth of the whole length. Frontally it curves downward and backward in front of the septum, and ends in the posteriorly tapering rostrum, which is connected below to the anterior commissure. The knee of the callosum is generally thicker than the body.

No constant difference was noted in the knee of the three groups, except that on the average in the Chinese it is a little rounder than in the European. In older brains its frontal pole may be more pointed, at least in my Dutch (in Fig. 2: ♀ 72, both ♀ 63, ♀ 60 with ♀ 17, ♂ 17 ♀ 25, both ♂ 34). More material would however be required to make sure whether this is an average fact.

The intermediate part of the callosum, the *truncus*, is slightly curved with the convexity upward and the grade of this curve differs in the various races, being more pronounced in my Chinese (Fig. 1) than in my Dutch brains (Fig. 2), as a glance at the figures shows.

This difference may be expressed in the height index of the callosum, divided by KAPPERS<sup>2)</sup>, being the greatest distance from the dorsum of the callosum to the *basal callosum line* (drawn in the figures) divided by the greatest distance between the most anterior and posterior point of

---

<sup>1)</sup> These brains, sent by the Central Institute for Brainresearch, Amsterdam, were from patients, who died (from non-nervous diseases) in the Wilhelmina gasthuis, Amsterdam.

<sup>2)</sup> ARIËNS KAPPERS. Indices for the anthropology of the brain, applied to Chinese, dolicho- and brachycephalic Dutch, foetus and neonati. These Proceedings Vol. 29, 1926.

the callosum. The index thus calculated for each brain is written in my figures (sex and age are mentioned in the left and right hand corner of each case).



From this results, that the callosum-height-index of my Dutch brains varies between 0.252 and 0.356, with an average of 0.300.

The callosum height-index in my Chinese varies between 0.307 and 0.438, with an average of 0.361.

While these figures are both a little less than those found by KAPPERS



Fig. 2. Thirty corpora callosa of adult Dutch. Average callosum height index 0.300.

in his Chinese and dolichocephalic Dutch brains<sup>1)</sup>, the difference is about the same in both series, KAPPERS finding 0.62, I 0.61 in favor of the Chinese.

*I may add that the callosum curve is still more striking in the Chinese foetus and in early child life than in the adult.*

The average callosum height index in my Philippino's was 0.340, being much nearer to that of my Chinese than to my Dutch brains.

This should not astonish us after KAPPERS' researches concerning the relationship between the hypsicephalic and brachycephalic character of the skull and the form of the callosum (l.c.), as KOEZE<sup>2)</sup> found the various races inhabiting the Philippino islands to be all hypsicephalic (with the exception only of the Igorrotes). Besides several of them, the Baluga's and Negrito's, are brachycephalic, whereas the others are meso- or subbrachycephalic (c.f. also SULLIVAN<sup>3)</sup> and BARROW<sup>4)</sup>).

Also the brain described by FALLOT<sup>5)</sup> was brachycephalic (83.3). Besides, according to DENIKER<sup>6)</sup>, they are considerably mixed with Chinese.

The diameter of the trunk also shows a certain degree of variation in the different races. Generally speaking, I have the impression that the body of the callosum is comparatively thinner in my Dutch brains than in my Chinese.

This may be due to a fronto-caudal compression which also appears in its steeper curve in the Chinese (the distance from the frontpole of the callosum to its hindpole showing an average of 7.47 cm in my Dutch and of 7.23 cm in my Chinese).

The trunk is rarely of even thickness throughout its length, generally its middle-part is thicker than the transition to the genu and to the splenium. At its transition to the splenium there may be even a more or less distinct groove, the *impressio corporis callosi*<sup>7)</sup>.

This groove is more often observed among the white people of my series than among either the Philippino or the Chinese. In fetal and early child life it fails or is only slightly indicated.

The splenium also shows considerable variation among the races studied. In my Dutch brains the splenium is either round or prolonged backward along the direction of the trunk.

1) Also from Amsterdam post mortems.

2) KOEZE. *Crania ethnica philippinica*, Veröffentlichungen des Niederländischen Reichsmuseums für Völkerkunde, Serie II N<sup>o</sup> 3, 1901—1904.

3) SULLIVAN'S researches seem to confirm KOEZE's findings, at least as far as concerns the length-breadth index (*Racial Types in the Philippine islands. Anthropological papers of the Amer. Museum of Nat. History*, Vol. 23, Part 1, 1918) and so do BARROW'S.

4) The negrito and allied types in the Philippines. *Amer. Anthropologist* Vol. 12, 1910, p. 358.

5) FALLOT. *Le cerveau d'un malais né à Manille*. *Bull. de la Société d'Anthropologie de Paris* 1880, p. 441.

6) DENIKER. *Les Races et les peuples de la terre*. Paris, 1910, p. 558.

7) ARIËNS KAPPERS. *Vergleichende Anatomie des Nervensystems der Wirbeltiere und des Menschen*. Vol. II, p. 1088. Fig. 572.

A pronounced downward curve of the splenium is rare among the Dutch and also among my Philippino's, (only in one case — second row N<sup>o</sup>. 3 —, it curves downward, but now forward).

In half of my Chinese, however, the splenium extends rather abruptly downwards, after which it may even turn forward.

It thus seems that the greater height-index of the callosum and the strong downward and eventual forward curve of the splenium may be considered an average racial distinction of the Chinese and that the first feature is also indicated — though in a less pronounced degree — in the Philippino's.

The *septum pellucidum*, extending between the inferior surface of the callosum and the superior surface of the fornix, consists of two layers, enclosing the so-called *ventriculus septi pellucidi*, a blind arachnoidal space, usually almost obliterated, but which at times may be rather extensive in size.

I found twice an enlargement of this space among the European and three times among Chinese.

In the case of one of the enlarged ventricles in the Chinese the 2 layers were separated from each other by about 2.5 cm and the anterior and posterior ends of the space are displaced, respectively forward and backward, apparently due to a hydropic condition.

The *septum pellucidum* may be divided for sake of description, into three parts, the anterior, the inferior and the posterior parts.

The *anterior portion* of the septum, situated immediately behind the posterior surface of the genu of the corpus callosum, is the largest. Its extension — specially its height — varies considerably in my Dutch brains, being larger in those where the genu is less pointed.



Fig. 3. Fifteen corpora callosa of adult Philippinos. Average callosum height index 0.340.



In my Philippinos this part of the septum shows no significant difference from the Dutch. In my Chinese, however, the anterior portion is generally higher (in both old and young brains) than in the Europeans or Philippinos, as may be expected from the greater rounding of the callosum.

The *inferior portion* of the septum situated between the posterior surface of the rostrum and the columnae anteriores fornicis is angular in shape, if the fornix and consequently the septum are not extended downward pathologically.

In both the Philippino and Chinese brains, the downward pointed angle of the inferior portion is more conspicuous than in the Dutch, i.e. the lower angle of the septum, which may be measured between a tangential on lowest parts of the columna anterior fornicis and the rostrum, on the average is sharper in the Chinese and Philippinos than in my Dutch brains.

It seems probable that this difference is correlated with the greater average curve and sagittal shortness of the Chinese and Philippino callosum.

The *posterior portion* of the septum — lying between the inferior surface of the posterior part of the callosum and the corresponding posterior part of the fornix — is generally very narrow and appears like a sharply pointed tail of the anterior portion, if the septum is not abnormally extended as in one case, that of the Dutch with a callosum-index of 0.314 ( $\sigma$ , ?) where the posterior portion has half the size of the anterior, and is prolonged as far as the free end of the splenium. Similar cases are my Chinese with a callosum index of 0.365 ( $\varphi$ , 78), 0.379 ( $\sigma$ , 39), 0.438 ( $\sigma$ , ?) and 0.341 ( $\sigma$ , 54).

It is most probable that this is due to a slight hydrops of the lateral ventricles which, as we know, not only extends the nervous ventricular wall, but also pushes the fornix downwards, thus increasing the extension of the septum, sometimes eroding it entirely<sup>1)</sup>.

In the Philippinos the posterior part of the septum shows practically no difference with the Dutch, being very small.

In most adult Chinese brains the posterior angle of the septum is generally a little wider, less acute than in either in the Dutch or Philippinos.

It may be that also this less sharp septum angle of the Chinese is due to the greater average curve of their callosum, by which the height of the septum is extended dorsally, also in its caudal division.

Finally I would remark that the course of the fornix — as far as can be judged from this material — is generally steeper in the Chinese and in the Philippino than in the Dutch brains here examined, a fact that may be also explained by the more hypsicephalic character of the Chinese and Philippino races.

<sup>1)</sup> So that both lateral ventricles freely communicate over the fornices.

From the above observations I conclude:

1. Among the Chinese the corpus callosum has a greater curve than in the Dutch. The average callosum index of the Philippinos is nearer to that of the Chinese than of the Dutch.

2. The impressio corporis callosi, occurs more frequently in my Dutch than in my Philippino or Chinese brains.

3. A downward bending of the splenium is most frequent in the Chinese.

4. The inferior angle of the septum (between the rostrum and the lower part of the columnae anteriores fornicis), frequently is less obtuse in the Chinese than in the Dutch, whereas the posterior angle of the septum is generally wider in the Chinese.

The fornix may have a steeper course in the Chinese and in the Philippino, compared to the Dutch.

---





